The Performance Loss of Unilateral Interference Cancellation

Luis Miguel Cortés-Peña, John R. Barry, and Douglas M. Blough
School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332–0250
{cortes, barry, doug.blough}@ece.gatech.edu

***Abstract*—We tackle the problem of determining the beamforming and combining weights in a network of interfering multiple-input multiple-output (MIMO) links. We classify any strategy for computing these weights as either *unilateral* or *bilateral*. A *unilateral* strategy is one for which the responsibility of cancelling interference from one node to another is preassigned to lie solely with only one of the two nodes, so that the other node is free to ignore the interference. Many existing strategies for managing interference in a network of MIMO nodes adopt the unilateral approach. In contrast, a *bilateral* strategy is one for which the responsibility of cancelling interference from one node to another is not preassigned, but is instead shared by both sides as the weights are computed. We present numerical examples to illustrate that bilateral strategies can significantly outperform unilateral strategies, especially for large networks and high interference. In one example, a bilateral approach delivers an aggregate capacity that is 227% higher than that of the best unilateral approach. We conclude that, although unilateral strategies are useful for determining whether or not the streams allocated in a network of MIMO links can coexist, the weight computation should be done bilaterally to prevent throughput loss.**

***Index Terms*—MIMO, degrees of freedom, beamforming, combining, weights, network**

I. INTRODUCTION

A multiple-input multiple-output (MIMO) link with n_r antennas at the receiver and n_t antennas at the transmitter can perform spatial multiplexing to potentially increase its capacity by a factor of $n = \min(n_r, n_t)$ without the need of additional spectrum or power [1]. Moreover, the performance improvement can be larger in multi-link networks when nodes use a combination of spatial multiplexing and interference cancellation [2, 3]. To achieve these performance gains, the network-wide beamforming and combining weights that support multiple streams and perform interference cancellation must be computed accordingly. We classify any strategy for computing these weights as either *unilateral* or *bilateral*.

We define an interference cancellation strategy as *unilateral* whenever the responsibility of cancelling interference is preassigned to either the transmitter or the receiver of this interference, but not both. Many existing strategies for managing interference in a network of MIMO links adopt a unilateral approach. Examples of unilateral strategies include the SRP/SRMP-CiM [4], SPACE-MAC [5], CAS [6], OBIC [7, 8], ExtendedGreedy [9], OSTM [10], and CLOM [11].

In contrast, we define an interference cancellation strategy as *bilateral* whenever the responsibility for cancelling interference is not preassigned to one of the two involved nodes, but is instead shared by both nodes in the process of determining the beamforming and combining weights. Examples of bilateral strategies include the SRMP-NiM [4], Max-SINR [12], IMMSE [13], incremental-SNR algorithm [14], and the MMSE and Max-SINR from [15].

A key distinction between the two classifications is that with unilateral interference cancellation, interference from one node to another can be ignored by one of the two nodes involved, whereas with bilateral interference cancellation, neither node can ignore this interference.

In this paper, we show that, for a three-link network, a proposed bilateral interference cancellation approach performs better than all known unilateral interference cancellation approaches, even after exhaustively searching for the best unilateral solution. For larger networks where an exhaustive search is infeasible, we use heuristics to search for the best possible unilateral and bilateral solution and we show that the sum capacity using bilateral interference cancellation can be significantly higher than the sum capacity using unilateral interference cancellation. We also show that by handling the cyclic interdependencies of the beamforming and combining weights, unilateral strategies can support a higher number of streams and achieve better performance.

This paper is organized as follows. In Section II, we define our model for the physical layer. In Section III, we provide a mathematical description of unilateral interference cancellation and the constraints necessary to determine feasibility of streams allocated in a network of MIMO links. In Section IV, we describe strategies for computing the beamforming and combining weights locally for a single node. In Section V, we present an algorithm that computes the beamforming and combining weights globally for every node. In Section VI, we present numerical results. Finally, in Section VII, we present our conclusions.

II. PHYSICAL-LAYER MODEL

Consider a set of M active links $L = \{(t^{[1]}, r^{[1]}), \ldots, (t^{[M]}, r^{[M]})\}$, where no node appears as an endpoint of more than one link, and $t^{[k]}$ and $r^{[k]}$ denote the transmitter and receiver of link k, respectively. Let $d^{[k]}$

be the number of multiplexed streams on link k, and let $n_t^{[k]}$ and $n_r^{[k]}$ be the number of antenna elements at $t^{[k]}$ and $r^{[k]}$, respectively. Let $\boldsymbol{H}^{[kl]} \in \mathbb{C}^{n_r^{[k]} \times n_t^{[l]}}$ be the matrix of complex channel gains between the antennas of $t^{[l]}$ and those of $r^{[k]}$. We assume that $E[|h_{i,j}^{[kl]}|^2] = 1$ for all i and j, where $h_{i,j}^{[kl]}$ is the element at the i^{th} row and j^{th} column of $\boldsymbol{H}^{[kl]}$.

The received vector after combining at $r^{[k]}$ is given by

$$\boldsymbol{y}^{[k]} = \boldsymbol{U}^{[k]\dagger} \sum_{l=1}^{M} \sqrt{\rho^{[kl]}} \boldsymbol{H}^{[kl]} \boldsymbol{V}^{[l]} \boldsymbol{x}^{[l]} + \boldsymbol{U}^{[k]\dagger} \boldsymbol{z}^{[k]}, \quad (1)$$

where $(\cdot)^\dagger$ is the conjugate transpose of $(\cdot)$; $\rho^{[kk]}$ is the signal-to-noise ratio (SNR) of link k; $\rho^{[kl]}$ for $l \neq k$ is the interference-to-noise ratio (INR) caused by $t^{[l]}$ at $r^{[k]}$; $\boldsymbol{V}^{[k]} \in \mathbb{C}^{n_t^{[k]} \times d^{[k]}}$ is the beamforming matrix of $t^{[k]}$; $\boldsymbol{U}^{[k]} \in \mathbb{C}^{n_r^{[k]} \times d^{[k]}}$ is the combining matrix of $r^{[k]}$; $\boldsymbol{x}^{[k]} \in \mathbb{C}^{d^{[k]}}$ is the transmit signal vector from $t^{[k]}$, assumed to be independently encoded Gaussian codebook symbols with unit-energy so that $E[\boldsymbol{x}^{[k]} \boldsymbol{x}^{[k]\dagger}] = \boldsymbol{I}_{d^{[k]}}$; and $\boldsymbol{z}^{[k]} \in \mathbb{C}^{n_r^{[k]}}$ is a vector of zero-mean circularly symmetric additive white Gaussian noise (AWGN) elements with unit variance. To satisfy the transmitter power constraint, the beamforming weights must satisfy $\mathrm{tr}\left(\boldsymbol{V}^{[k]} \boldsymbol{V}^{[k]\dagger}\right) = 1$, for all $k \in \{1, \ldots, M\}$.

The signal-to-interference-plus-noise ratio (SINR) of stream i in link k is then given by [16]

$$\mathrm{SINR}^{[ki]} = \frac{\gamma^{[ki]}}{\boldsymbol{U}_{*i}^{[k]\dagger} \boldsymbol{B}^{[k]} \boldsymbol{U}_{*i}^{[k]} - \gamma^{[ki]}}, \quad (2)$$

where $(\cdot)_{*i}$ is the i^{th} column of $(\cdot)$ and

$$\gamma^{[ki]} = \rho^{[kk]} \boldsymbol{U}_{*i}^{[k]\dagger} \boldsymbol{H}^{[kk]} \boldsymbol{V}_{*i}^{[k]} \boldsymbol{V}_{*i}^{[k]\dagger} \boldsymbol{H}^{[kk]\dagger} \boldsymbol{U}_{*i}^{[k]}, \quad (3)$$

$$\boldsymbol{B}^{[k]} = \boldsymbol{I}_{n_r^{[k]}} + \sum_{l=1}^{M} \rho^{[kl]} \boldsymbol{H}^{[kl]} \boldsymbol{V}^{[l]} \boldsymbol{V}^{[l]\dagger} \boldsymbol{H}^{[kl]\dagger}. \quad (4)$$

Finally, the instantaneous capacity in bits/sec/Hz of link k is

$$C^{[k]} = \sum_{i=1}^{d^{[k]}} \log_2\left(1 + \mathrm{SINR}^{[ki]}\right). \quad (5)$$

III. Unilateral Interference Cancellation and Feasibility

It will be convenient to characterize a unilateral interference cancellation strategy by a pair of matrices: one ($\boldsymbol{A}^t$) for the transmitters, and one ($\boldsymbol{A}^r$) for the receivers. These matrices contain the *interference cancellation assignment* that specifies, for each node, which nodes' interference must be cancelled and which nodes' interference can be safely ignored. The entry $a_{l,k}^t \in \{0,1\}$ at the l^{th} row and the k^{th} column of $\boldsymbol{A}^t$ is one if $t^{[l]}$ is assigned to cancel its interference at $r^{[k]}$ and zero otherwise. Similarly $a_{l,k}^r \in \{0,1\}$ is one if $r^{[l]}$ is assigned to cancel the interference from $t^{[k]}$ and zero otherwise.

The entries for $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$ are set according to the following rules. We set $a_{k,l}^r = a_{l,k}^t = 0$ if $\rho^{[kl]} < \varrho$ for some threshold ϱ. For $\rho^{[kl]} \geq \varrho$ and $k \neq l$, however, interference is either cancelled at the transmitter or the receiver, but not both, and so $a_{k,l}^r = 1 - a_{l,k}^t$. Finally, for convenience, we define $a_{k,k}^r = a_{k,k}^t = 1$ for all $k \in \{1, \ldots, M\}$.

For a unilateral interference cancellation strategy, we define a *stream allocation* $\boldsymbol{d} = [d^{[1]}, d^{[2]}, \ldots, d^{[M]}]$ as *feasible* if and only if there exist interference cancellation assignment matrices $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$ as defined above such that the degrees-of-freedom constraints $\sum_{l=1}^{M} a_{k,l}^t d^{[l]} \leq n_t^{[k]}$ and $\sum_{l=1}^{M} a_{k,l}^r d^{[l]} \leq n_r^{[k]}$ are satisfied for all $k \in \{1, \ldots, M\}$.

IV. Locally Calculating the Beamforming and Combining Weights

The computation of weights in a network is complicated by the fact that the transmitter beamforming weights and receiver combining weights are interdependent: the beamforming weights that cancel interference depend on the corresponding combining weights, while the combining weights that cancel interference depend on the corresponding beamforming weights. In Section V, we propose an iterative algorithm that deals with this problem. For now, as a stepping stone, we show in this section how to compute the combining weights as a function of the relevant beamforming weights, and how to compute the beamforming weights as a function of the relevant combining weights.

For convenience we normalize the combining weights at $r^{[k]}$ according to

$$\boldsymbol{U}^{[k]} = \hat{\boldsymbol{W}}^{[k]} \sqrt{\frac{1}{d^{[k]}}}, \quad (6)$$

where $\hat{\boldsymbol{W}}^{[k]}$ is the matrix formed after dividing each column vector of $\boldsymbol{W}^{[k]}$ with its corresponding Euclidean norm. In the following, we specify different ways of computing $\boldsymbol{W}^{[k]}$.

A. Zero-Forcing Combining for Unilateral Cancellation

The zero-forcing (ZF) combining weights eliminate all interference, despite the penalty of reducing its signal energy [17]. In the context of a unilateral strategy, for which the cancellation responsibilities are preassigned according to $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$, the ZF combining weights at $r^{[k]}$ are

$$\boldsymbol{W}_{*i}^{[k]} = \boldsymbol{h}_i^{[k]} - \boldsymbol{\hbar}_i^{[k]}, \quad (7)$$

for all $i \in \{1, \cdots, d^{[k]}\}$, where $\boldsymbol{h}_i^{[k]} = \boldsymbol{H}^{[kk]} \boldsymbol{V}_{*i}^{[k]}$, and $\boldsymbol{\hbar}_i^{[k]}$ is the projection of $\boldsymbol{h}_i^{[k]}$ onto the span of all columns of $\boldsymbol{h}_{j \neq i}^{[k]}$ and all columns of $a_{k,l}^r \boldsymbol{H}^{[kl]} \boldsymbol{V}^{[l]}$ for $l \neq k$. Notice that these ZF weights eliminate interference from node l only if $a_{k,l}^r = 1$, and ignores it otherwise.

B. Minimum Mean-Squared-Error Combining for Unilateral and Bilateral Cancellation

A minimum mean-squared-error (MMSE) receiver relaxes the zero interference constraint with the advantage of allowing more signal to be collected [17]. In the context of a unilateral strategy, for which the cancellation responsibilities are preassigned according to $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$, the MMSE combining weights at $r^{[k]}$ are

$$\boldsymbol{W}^{[k]} = \left(\boldsymbol{R}^{[k]} + \boldsymbol{I}_{n_r^{[k]}}\right)^{-1} \boldsymbol{H}^{[kk]} \boldsymbol{V}^{[k]}, \quad (8)$$

where

$$\boldsymbol{R}^{[k]} = \sum_{l=1}^{M} a^{r}_{k,l} \rho^{[kl]} \boldsymbol{H}^{[kl]} \boldsymbol{V}^{[l]} \boldsymbol{V}^{[l]\dagger} \boldsymbol{H}^{[kl]\dagger}. \qquad (9)$$

The presence of $a^{r}_{k,l}$ in (9) ensures that these weights cancel only the interference that they are assigned to cancel.

MMSE can also be defined in the context of bilateral interference cancellation. To do so, we set $a^{t}_{l,k} = a^{r}_{k,l} = 1$ for $\rho^{[kl]} \geq \varrho$ and $k \neq l$, i.e. both $r^{[k]}$ and $t^{[l]}$ include this interference in the computation of their weights. With this modification, we can use (8) and (9) to compute the MMSE combining weights at $r^{[k]}$ in the context of bilateral interference cancellation.

The MMSE weights for bilateral interference cancellation are equal to the Max-SINR weights from [12] since weights that minimize the mean-squared-error also maximize the SINR [16]. We prefer to use MMSE weights instead of Max-SINR weights because MMSE weights can be computed with lower complexity than Max-SINR weights, since all weight columns of MMSE can be computed after one matrix inversion instead of computing a matrix inversion for each weight column of Max-SINR.

C. Beamforming Via a Virtual Network

To compute the beamforming weights for a transmitter that performs interference cancellation, we follow [18] and reverse the channel, creating a virtual network in which we compute the beamforming weights assuming the transmitter is a virtual receiver. Specifically, to compute the beamforming weights, add a $\leftarrow$ to all variables in (6) to (9), then compute $\overleftarrow{\boldsymbol{H}}^{[lk]} = \boldsymbol{H}^{[kl]\dagger}$, $\overleftarrow{\boldsymbol{V}}^{[k]} = \boldsymbol{U}^{[k]}$, $\overleftarrow{\boldsymbol{U}}^{[k]} = \boldsymbol{V}^{[k]}$, and $\overleftarrow{a}^{r}_{k,l} = a^{t}_{k,l}$. The resulting beamforming weights allocate equal power to each stream since $\boldsymbol{V}^{[k]\dagger}_{*i} \boldsymbol{V}^{[k]}_{*i} = \overleftarrow{\boldsymbol{U}}^{[k]\dagger}_{*i} \overleftarrow{\boldsymbol{U}}^{[k]}_{*i} = \frac{1}{d^{[k]}}$ for all $i \in \{1, \ldots, d^{[k]}\}$.

The combination of this virtual procedure, and the MMSE receiver weights (8) results in a set of beamforming weights that we will loosely refer to as MMSE, even though strictly speaking they do not minimize the sum mean-squared error at the receivers.

V. Globally Calculating the Beamforming and Combining Weights

The interdependency of the beamforming and combining weights can create dependency cycles that significantly complicate their optimization. For example, consider the three link example shown in Figure 1. For a unilateral interference cancellation strategy, there are two interference cancellation assignments in which two streams per link are feasible. Figure 1 depicts one of the two interference cancellation assignments, namely

$$\boldsymbol{A}^{t} = \boldsymbol{A}^{r} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}. \qquad (10)$$

The other assignment can be obtained by transposing (10). Given (10), the ZF and MMSE weights of $r^{[1]}$ are dependent on the weights at $t^{[1]}$ and $t^{[3]}$. Furthermore, $t^{[3]}$'s weights depend on the weights of $r^{[3]}$ and $r^{[2]}$. If the nulling assignment is followed, this sequence traverses every node and completes a cycle when calculating the weights of $t^{[2]}$, which depend on the weights of $r^{[1]}$, the initial node.

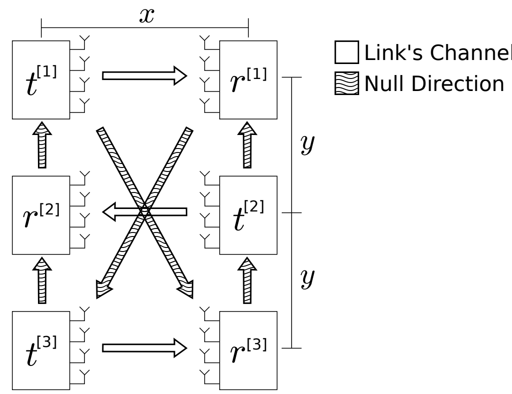

Fig. 1. Topology of simulated three-link network. At least one high interfering node is located at a distance y from every receiver.

Our solution to the problem of dependency cycles is to iteratively compute the beamforming and combining weights. Similar iterative approaches were taken in the previous reported bilateral algorithms of [12–15].

Our algorithm, called ComputeWeights, computes the weights of every node in a network by initializing the beamforming and combining weights according to the highest eigenmodes of the desired channel's singular value decomposition (SVD) [1] and computing the interference cancellation weights iteratively until the weights converge (within a threshold ϵ) or a maximum number of iterations N_{max} is reached.

For convenience, we define Algorithm ComputeWeights as a general algorithm so that we can reuse it in the context of unilateral or bilateral interference cancellation. As we will show in the next section, the inputs of ComputeWeights vary depending on the interference cancellation strategy. The possible inputs for ComputeWeights are: the set of all channels $\{\boldsymbol{H}\}$; the set of all SNRs and INRs $\{\rho\}$; the interference cancellation assignment matrices $\boldsymbol{A}^{t}$ and $\boldsymbol{A}^{r}$; a stream allocation $\boldsymbol{d}$; a *node schedule* $\boldsymbol{s}$ that defines the order in which weights are computed (important since different node orderings produce different results); and a flag F that indicates ZF ($F = 0$) or MMSE ($F = 1$) weights.

Next, we describe two unilateral interference cancellation algorithms and a bilateral interference cancellation algorithm that compute the particular weights of all links based on Algorithm ComputeWeights.

A. Global Weights for Unilateral Interference Cancellation

We use ComputeWeights to create two instances that can compute the unilateral interference cancellation weights, namely ComputeWeights with ZF unilateral interference cancellation, and ComputeWeights with MMSE unilateral interference cancellation. ComputeWeights with ZF unilateral interference cancellation can be obtained using the inputs

Algorithm ComputeWeights: Algorithm for computing the weights of each node.

Input: $(\{\boldsymbol{H}\}, \{\rho\}, \boldsymbol{A}^t, \boldsymbol{A}^r, \boldsymbol{d}, \boldsymbol{s}, F)$
Output: Beamforming and combining weights of each node in the network.

for each *link* k **do**
  Initialize $r^{[k]}$'s and $t^{[k]}$'s weights to link k's SVD corresponding to the highest $d^{[k]}$ eigenmodes;
  if *link* k *performs interference cancellation* **then**
    Allocate equal power $\sqrt{\frac{1}{d^{[k]}}}$;
    Remove any node in link k from $\boldsymbol{s}$ that does not perform interference cancellation;
  else
    Allocate optimal power on the $d^{[k]}$ highest eigenmodes via waterfilling;
    Remove $t^{[k]}$ and $r^{[k]}$ from $\boldsymbol{s}$;
  end
end
for $iteration \leftarrow 1$ **to** N_{max} **do**
  for each s_i *in increasing* i **do**
    Set k equal to the link number of node s_i;
    if s_i *is a receiver* **then**
      Compute $r^{[k]}$'s weights using (7) if $F = 0$, or (8) and (9) if $F = 1$;
    else s_i is a transmitter
      Reverse the communication link ;
      Compute $\overleftarrow{r}^{[k]}$'s weights using (7) if $F = 0$, or (8) and (9) if $F = 1$;
    end
  end
  Stop if the maximum absolute value of the difference of elements between the previous weights and the newly computed weighs is less than ϵ for all s_i;
end

$(\{\boldsymbol{H}\}, \{\rho\}, \boldsymbol{A}^t, \boldsymbol{A}^r, \boldsymbol{d}, \boldsymbol{s}, F = 0)$, and ComputeWeights with MMSE unilateral interference cancellation can be obtained using the inputs $(\{\boldsymbol{H}\}, \{\rho\}, \boldsymbol{A}^t, \boldsymbol{A}^r, \boldsymbol{d}, \boldsymbol{s}, F = 1)$. Here, $\boldsymbol{d}$ must be feasible, and $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$ must be the corresponding interference cancellation assignment.

B. OBIC: A Cycle-Free Unilateral Strategy

Another unilateral strategy is the ***order-based interference cancellation (OBIC)*** strategy from [7, 8], which is based on the rule that nodes being scheduled must cancel interference from previously scheduled interfering nodes. OBIC is inherently unilateral since each node ignores interference to and from all nodes that are scheduled after it. The entries of $\boldsymbol{A}^r$ and $\boldsymbol{A}^t$ are populated as nodes are scheduled. We define a node schedule as *feasible under OBIC* if for every scheduled node, the node can cancel the interference from all previously scheduled nodes without violating the degrees-of-freedom constraints.

OBIC will not generally consider all stream allocations that are feasible. For example, in the context of the three-link example in Figure 1, the stream allocation $\boldsymbol{d} = [2, 2, 2]$ is not feasible under OBIC. The reason is that the OBIC scheduling mechanism specifically excludes cycles.

In [7], the authors propose that interference cancellation under OBIC be done with ZF. However, we also define an MMSE-based OBIC strategy called ***OBICmmse*** that uses MMSE instead of ZF to perform interference cancellation on the previously scheduled interfering nodes. We implement OBIC using ComputeWeights with the inputs $(\{\boldsymbol{H}\}, \{\rho\}, \boldsymbol{A}^t, \boldsymbol{A}^r, \boldsymbol{d}, \boldsymbol{s}, F = 0)$ and OBICmmse using ComputeWeights with inputs $(\{\boldsymbol{H}\}, \{\rho\}, \boldsymbol{A}^t, \boldsymbol{A}^r, \boldsymbol{d}, \boldsymbol{s}, F = 1)$. These instances of ComputeWeights execute a single iteration ($N_{max} = 1$). Also, the input node schedule $\boldsymbol{s}$ must be feasible under OBIC and it defines $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$.

C. Global Weights for Bilateral Interference Cancellation

We reuse Algorithm ComputeWeights to iteratively compute the beamforming and combining weights for the case of bilateral interference cancellation. For this instance of ComputeWeights, we fix the node schedule to $\boldsymbol{s}_\circ = [r^{[1]}, r^{[2]}, \ldots, r^{[M]}, t^{[1]}, t^{[2]}, \ldots, t^{[M]}]$, i.e., all receivers first, followed by all transmitters. We obtain ComputeWeights with bilateral interference cancellation using the inputs $(\{\boldsymbol{H}\}, \{\rho\}, \boldsymbol{A}^t, \boldsymbol{A}^r, \boldsymbol{d}, \boldsymbol{s} = \boldsymbol{s}_\circ, F = 1)$. For this instance of ComputeWeights, the definition of $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$ are modified for bilateral interference cancellation, so that $a^t_{l,k} = a^r_{k,l} = 1$ for $\rho^{[kl]} \geq \varrho$.

VI. Numerical Results

This section is organized as follows. In Section VI-A, we present a specific example where the performance of bilateral interference cancellation is significantly better than that of the best unilateral interference cancellation. In Sections VI-B and VI-C, we present results comparing the bilateral versus the unilateral strategies in a three-link network and in a random eight link network, respectively. In section VI-D, we show the advantage of overcoming dependency cycles over avoiding them for unilateral interference cancellation. For all simulations, we fix $\varrho = -2.9$ dB, we fix $\epsilon = 0.0001$, we set the reference SNR and INR at one meter to 57.1 dB, and unless otherwise stated, the SNR and INR vary inversely proportional to the distance cubed. Except for the example on section VI-A, we assume a "quasi-static" flat-fading Rayleigh model where the channel is assumed constant for the duration of a burst, but random between bursts, and the channel elements are independent and identically distributed, complex Gaussian with zero mean and unit variance [19].

A. An Example

Consider three links where each node has two antenna elements spaced at half-wavelength and each link carries a single stream. For this example only, we consider a channel without fading. We locate the transmitters and receivers as shown in Figure 1 with $y = 25$, and $x = 50$. For $t^{[1]}$,

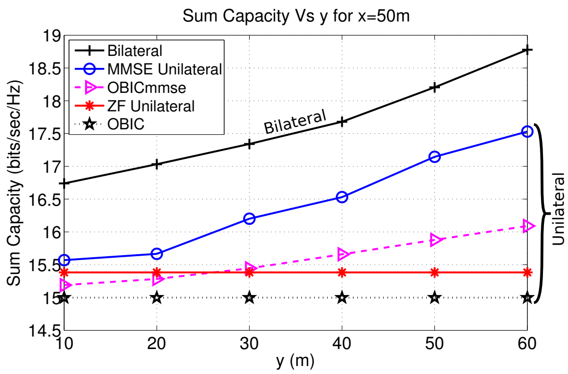

Fig. 2. Maximum capacity of all stream allocations, node schedules, and interference cancellation assignments for the three link network of Figure 1.

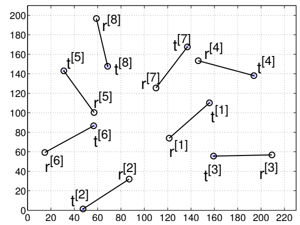

Fig. 3. Topology of simulated eight link network.

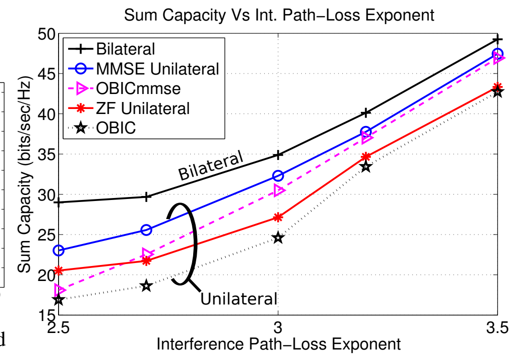

Fig. 4. Plot of maximum sum capacity versus interference path-loss exponent α_I for fixed signal path-loss exponent $\alpha_S = 3$.

$t^{[2]}$, and $t^{[3]}$, the angles as measured counterclockwise from the horizontal axis to the line through the two antennas are $131.1°$, $136°$, and $29.8°$, respectively. For $r^{[1]}$, $r^{[2]}$, and $r^{[3]}$, these angles are $23.4°$, $134.1°$, and $135°$, respectively. Taking $t^{[3]}$ as the origin, we place a reflector with 0.9 attenuation at $(25, 70)$. We set $N_{max} = 1000$, and exhaustively search for the node schedule and interference cancellation assignment that produces the highest aggregate capacity for the unilateral approaches. In this scenario, the sum capacity using ComputeWeights with bilateral interference cancellation is 10.22 bits/sec/Hz, while the sum capacity using the best unilateral strategy (ComputeWeights with MMSE unilateral interference cancellation) is 3.12 bits/sec/Hz. The bilateral strategy thus outperforms the best unilateral strategy by 227%.

B. Three-Link Results

Unlike the previous Section VI-A, we return to Rayleigh fading. We consider the three-link example of Figure 1, where every node has four antennas. For each link, we allocate zero to four streams. Where applicable, we calculate weights for all stream allocations, interference cancellation assignments, and node schedules. We set $N_{max} = 1000$, and we record only the highest capacity of every interference cancellation assignment and node schedule that converge.

We show how the aggregate capacity varies with interference for this three-link example. Figure 2 shows the maximum capacity of all stream allocations, averaged over 100 trials, plotted as a function of the distance y for fixed $x = 50$. As can be observed, as interference decreases (y increases), the capacities of ComputeWeights with bilateral interference cancellation, ComputeWeights with MMSE unilateral interference cancellation, and OBICmmse increase. The slope, however, is larger for ComputeWeights with bilateral interference cancellation and ComputeWeights with MMSE unilateral interference cancellation than for OBICmmse. The sum capacity of ComputeWeights with MMSE unilateral interference cancellation is between 5.8% and 8% less than the sum capacity of ComputeWeights with bilateral interference cancellation for all values of y. Figure 2 also shows that the OBIC, ComputeWeights with ZF unilateral interference cancellation, OBICmmse, and ComputeWeights with MMSE unilateral interference cancelation strategies had at worst a 20%, 18%, 14%, and 8% capacity loss as compared to ComputeWeights with bilateral interference cancellation.

C. Larger Network Results

We now present results for the eight MIMO links shown in Figure 3 in which each receiver is 50 meters from its transmitter. We fix all nodes to have four antenna elements, and we allocate zero to four streams at each link. For such a network size, the computation time required to test all possible stream allocations, all possible node schedules, and all possible interference cancellation assignments is excessive. For this reason, we use the feasibility heuristic ExtendedGreedy from [9] to find a stream allocation space that is feasible. We use $\boldsymbol{A}^t$ and $\boldsymbol{A}^r$ provided by ExtendedGreedy as input to ComputeWeights with ZF/MMSE unilateral interference cancellation. We heuristically determine the node schedule for ComputeWeights with ZF/MMSE unilateral interference cancellation by scheduling nodes that depend the least on other nodes first. For OBIC, we average the results over a maximum of five random OBIC feasible node schedules for each stream allocation. We fix $N_{max} = 10000$ and we only record data for stream allocations that converge.

Let α_S be the desired signal's path-loss exponent, and α_I be the path-loss exponent between every interfering transmitter-receiver pair. We fix $\alpha_S = 3$ and vary α_I to vary the interference. We let $\varrho = -2.9$ dB and so $\alpha_I = 2.5$, $\alpha_I = 2.7$, $\alpha_I = 3$, $\alpha_I = 3.2$, and $\alpha_I = 3.5$ corresponds to 100%, 87%, 48%, 21%, and 13% of all interference satisfying $\rho^{[kl]} \geq \varrho$, respectively.

Figure 4 depicts the maximum sum capacity as a function of α_I averaged over 50 random channel realizations. OBIC based strategies performed poorly at high interference ($\alpha_I = 2.5$) possibly due to their limited stream allocation space. ComputeWeights with bilateral interference cancellation outperformed the best unilateral interference cancellation method (ComputeWeights with MMSE unilateral interference cancellation) by 26%, 8%, and 4% at high, medium, and low interference, respectively. Also, ComputeWeights with bilateral interference cancellation outperformed OBIC and OBICmmse by 71% and 60%, 42% and 14%, and 15% and 5% at high, medium, and low interference, respectively. It is possible that other stream allocations exist in which ComputeWeights with bilateral interference cancellation performs better than that of the results shown in Figure 4 since we have constrained the stream allocation space to be feasible. Clearly, Figure 4 shows that deviating from ComputeWeights with bilateral interference cancellation in large networks, where many links are scheduled concurrently, can result in large penalties in the aggregate throughput.

D. The Advantage of Overcoming Cycles

Using the same simulation setup from Section VI-B, we now look at how the aggregate capacity varies with different stream allocations to show the benefit of overcoming cycles. We compare only between ComputeWeights with ZF/MMSE unilateral interference cancellation and OBIC/OBICmmse, but we also show results for ComputeWeights with bilateral interference cancellation. Figure 5 shows the average capacity of 100 random channel realizations for the most relevant stream allocations and $x = y = 50$. The stream allocations $\boldsymbol{d} = [1,1,2]$, $\boldsymbol{d} = [1,2,1]$, $\boldsymbol{d} = [2,1,2]$, and $\boldsymbol{d} = [2,2,2]$ are the allocations in which OBIC/OBICmmse, ComputeWeights with ZF unilateral interference cancellation, ComputeWeights with MMSE unilateral interference cancellation, and ComputeWeights with bilateral interference cancellation achieved the highest capacity, respectively. Notice that for $\boldsymbol{d} = [2,1,2]$ and $\boldsymbol{d} = [2,2,2]$ an OBIC feasible node schedule does not exist (cycles cannot be avoided) and so we show no results for OBIC/OBICmmse for these allocations. Figure 5 shows that ComputeWeights with MMSE unilateral interference cancellation for $\boldsymbol{d} = [2,1,2]$ outperforms the best of OBIC and OBICmmse by 14% and 8%, respectively. For a unilateral interference cancellation strategy, these results show that overcoming cycles results in higher sum capacity than avoiding cycles because more streams can be allocated per link when cycles are present.

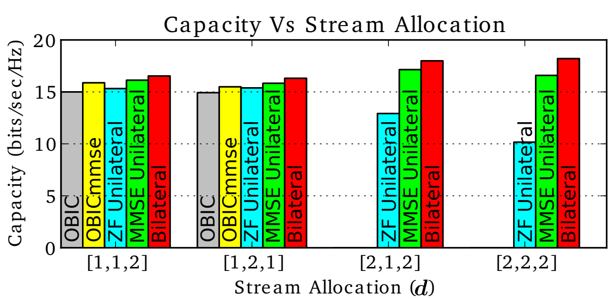

Fig. 5. Capacities of most relevant stream allocations for the three-link topology of Figure 1.

VII. Conclusion

We showed that, for a three-link example, a bilateral interference cancellation strategy performs better than the best unilateral interference cancellation strategy even after considering all node schedules and all interference cancellation assignments for the unilateral interference cancellation strategy. We showed that the performance loss of unilateral strategies can be greater in larger networks. Using the three-link example, we showed that overcoming dependency cycles leads to a higher number of streams in the network than preventing cycles, which improves the performance of the network. We conclude that while the unilateral interference cancellation strategy can aid network designers in determining the feasibility of a stream allocation in the network, it is ultimately the weight algorithm that determines the performance of the network, so the weights should be computed bilaterally to find the best weights for network operation.

References

[1] A. Molisch, *Wireless Communications*. Wiley-IEEE Press, 2005.

[2] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the k user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[3] R. Srinivasan, D. Blough, and P. Santi, "Optimal one-shot stream scheduling for MIMO links in a single collision domain," in *Proc. IEEE Secon*, Jun. 2009, pp. 1–9.

[4] B. Hamdaoui and K. Shin, "Characterization and analysis of multi-hop wireless MIMO network throughput," in *ACM MobiHoc*, Sep. 2007, pp. 120–129.

[5] J. Park, A. Nandan, M. Gerla, and H. Lee, "SPACE-MAC: Enabling spatial reuse using MIMO channel-aware MAC," in *Proc. IEEE ICC*, vol. 5, May 2005, pp. 3642–3646.

[6] K. Sundaresan, W. Wang, and S. Eidenbenz, "Algorithmic aspects of communication in ad-hoc networks with smart antennas," in *ACM MobiHoc*, May 2006, pp. 298–309.

[7] J. Liu, Y. Shi, and Y. Hou, "A tractable and accurate cross-layer model for multi-hop MIMO networks," in *Proc. IEEE Infocom*, Mar. 2010, pp. 1–9.

[8] Y. Shi, J. Liu, C. Jiang, C. Gao, and Y. Hou, "An optimal link layer model for multi-hop MIMO networks," in *Proc. IEEE Infocom*, Apr. 2011, pp. 1916–1924.

[9] R. Srinivasan, D. Blough, L. Cortés-Peña, and P. Santi, "Maximizing throughput in MIMO networks with variable rate streams," in *European Wireless Conf.*, Apr. 2010, pp. 551–559.

[10] D. Blough, G. Resta, P. Santi, R. Shrinivasan, and L. Cortés-Peña, "Optimal one-shot scheduling for MIMO networks," in *Proc. IEEE Secon*, Jun. 2011, pp. 404–412.

[11] R. Bhatia and L. Li, "Throughput optimization of wireless mesh networks with MIMO links," in *Proc. IEEE Infocom*, May 2007, pp. 2326–2330.

[12] K. Gomadam, V. Cadambe, and S. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *Proc. IEEE Globecom*, Dec. 2008, pp. 1–6.

[13] R. Iltis, S. Kim, and D. Hoang, "Noncooperative iterative MMSE beamforming algorithms for ad hoc networks," *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 748–759, Apr. 2006.

[14] D. Schmidt, W. Utschick, and M. Honig, "Beamforming techniques for single-beam MIMO interference networks," in *Allerton Conf. Commun., Control and Comput.*, Oct. 2010, pp. 1182–1187.

[15] S. Peters and R. Heath, "Cooperative algorithms for MIMO interference channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 1, pp. 206–218, Jan. 2011.

[16] J. Choi, *Optimal Combining and Detection: Statistical Signal Processing for Communications*. Cambridge University Press, 2010.

[17] J. Barry, E. Lee, and D. Messerschmitt, *Digital Communication*. Springer Netherlands, 2004.

[18] F. Rashid-Farrokhi, K. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1437–1450, Oct. 1998.

[19] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.