

Interference-Aware Multicast for Wireless Multihop Networks

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Abstract—In this paper, we revisit the problem of optimal tree-based routing structures for multicast in wireless multihop networks, but accounting for the impact of interference. Our analysis is based on the most accurate known interference model, namely the SINR-based physical interference model. We first study the problem in a low-intensity multicast scenario where we derive optimal node selection strategies for different subtree structures. We then extend these analyses to account for interference between consecutive packets in higher-rate multicast scenarios. Based on these analyses, we propose and evaluate two new multicast routing structures: the interference-aware Steiner tree (IAST) algorithm, which requires global knowledge of node locations, and the fixed-distance tree merging (FTM) algorithm, which does not require global knowledge. We show that our proposed algorithms provide up to 57% reduction in schedule length and up to 41% increase in goodput over existing tree-based routing structures.

I. INTRODUCTION

In multicast, a single message is delivered to a group of destinations in a network. This problem has been studied for both wired and wireless networks. A survey of multicast protocols for ad hoc networks can be found in [1]. A major limitation of research in this area, to date, is that the vast majority of works ignore interference, which is a significant factor in wireless multihop networks. The few works that do consider interference use inaccurate models. In this paper, we carry out the first research study of end-to-end multicast routing structures for wireless multihop networks that accounts for interference using accurate interference models. We design new interference-aware multicast routing structures and show that their performance substantially exceeds that of existing multicast algorithms that do not account for interference.

Multicast routing approaches can be classified into three main categories: tree-based, mesh-based, and structure-less. Tree-based protocols [2], [3], [4], [5] use different kinds of trees as underlying routing structure to route multicast messages to all destinations. Tree structures provide simple and cost effective routing infrastructures at the cost of robustness in the presence of mobility and link failures. Mesh-based protocols [6], [7], [8] use mesh structures to provide robustness by having multiple routes between the source and destinations at the cost of mesh structure maintenance. Structure-less

multicast protocols do not explicitly create a routing structure but rely on other methods such as network coding [9], [10] and geographic routing [11], [12]. In this initial study of interference-aware multicast, we focus on tree-based protocols due to their simplicity and cost effectiveness. Extending the analyses and concepts developed herein to mesh-based protocols is an interesting topic for future research.

Most tree-based protocols have been based on shortest path trees or Steiner trees. The goal of shortest path trees (e.g. [3], [4]) is to minimize the distance between the source and each destination, while the goal of Steiner trees (e.g. [13], [14]) is to minimize the sum of the distances in the multicast tree. A few studies comparing these predominant tree structures have been done. Ruiz and Gomez-Skarmeta [15] studied shortest path trees and Steiner trees. The authors argued that Steiner tree is not appropriate for wireless networks and proposed that the problem should be re-formulated to minimize the cost in terms of the number of forwarding nodes. They proposed a greedy heuristic algorithm, called MNT, and showed that the proposed algorithm is able to reduce the number of forwarding nodes. Nguyen [16] revisited the study and evaluated the performance of shortest path trees, Steiner trees, and the MNT algorithm in terms of packet delivery ratio. The author showed that shortest path trees offer the best performance in terms of packet delivery ratio. However, neither of these studies accounted for interference in their evaluations.

Other work that studied multicast scaling law and structure are [17], [18], [19]. The authors studied the asymptotic multicast capacity of multihop wireless networks. In [19], a comb structure for multicast trees that achieves capacity in the order sense was proposed. While the work did account for interference, they used the protocol model for interference, which is not as accurate as the physical interference model, and they were concerned primarily with asymptotic scaling results, rather than best performance on finite networks.

A number of studies consider the multicast problem with different goals such as energy [20], [21], [22], cost of building and maintaining multicast trees [23]. We do not study these issues in our paper.

Scheduling is an important aspect of wireless multihop networks with interference. Scheduling can increase network throughput by letting devices access the channel in an orderly

fashion instead of the more conservative contention-based access. The only multicast scheduling works of which we are aware are [24], [25]. However, [24] is primarily concerned with power control and scheduling plays only a minor role, while [25] only deals with one-hop (not end-to-end) multicast.

In this paper, we consider the problem of interference-aware multicast routing tree in wireless multihop networks, using an accurate physical interference model. First, we study the problem for low-intensity multicast. We classify nodes into different classes and derive optimal interference-aware routing strategies for each class. Based on those analyses, we propose a new multicast routing structure based on Steiner trees for the low-intensity case. Next, we consider a general multicast scenario and propose a second new multicast routing structure that is not based on Steiner tree. We evaluate our proposed structures through simulation in both the TDMA and CSMA/CA settings. Simulation results demonstrate that, compared to existing approaches, our proposed structures reduce the schedule length up to 57% in TDMA networks and achieve up to 41% higher goodput in CSMA/CA networks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a communication graph $G = (V, E)$, where V is a set of all wireless nodes. An edge $(u, v) \in E$ if and only if $d(u, v) \leq r_t$, where $d(u, v)$ is the Euclidean distance between nodes u and v and r_t is the maximum transmission range. We are given a multicast source $s \in V$ and a set of multicast destinations $M \subset V$. The problem is to construct a multicast tree rooted at s that spans M , along with a partition of a set of non-leaf nodes in the multicast tree S_1, S_2, \dots, S_k , where S_i is a set of feasible transmissions with the given interference model. A feasible transmission set is a set where, if all nodes in the set transmit to their respective receivers at the same time, all receivers will successfully receive the transmissions. We call the partition S_1, S_2, \dots, S_k a schedule with schedule length k . Our goal is to construct a minimum length schedule.

We adopt the classical model for radio signal propagation, which is referred to as the log-distance propagation loss model. The radio signal strength at a distance d from the transmitter is given by $\frac{P}{d^\alpha}$, where P is the transmission power and α is the path loss coefficient. We assume that nodes are not equipped with interference cancellation capabilities.

In this paper, we consider the physical interference (PI) model [26]. In the PI model, interference from all concurrent transmitters in the network, no matter how distant, is factored into the signal-to-interference ratio (SIR) value at the receiver. Specifically, the transmission will be correctly received by the receiver if and only if the SIR value at the receiver is larger than the SIR threshold (SIR_{\min}).

III. INTERFERENCE-AWARE ROUTING FOR LOW INTENSITY MULTICAST

In this section, we consider low intensity multicast, which is defined as a situation where the time between two consecutive packets generated by the source is greater than the time it takes to deliver one multicast packet to all destinations. We

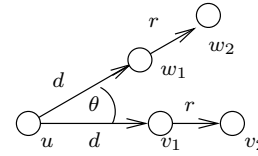


Fig. 1. A branching node u branching into two paths.

first consider an ideal network where we are given a source node s and a set of destination nodes M and we are able to place extra nodes anywhere in the network when building a multicast tree. Our goal is to construct a tree rooted at s that spans M while taking interference into account. To achieve this, we classify nodes in a multicast tree into three classes.

- Leaf nodes – nodes with no child.
- Path nodes – nodes with exactly one child.
- Branching nodes – nodes with more than one child.

According to our classification, leaf nodes do not forward data in the multicast tree and so they do not generate interference. Furthermore, along a single path, there is no interference between nodes since there is only one packet being transmitted at a time. Thus, the optimal structure along a single path is for each transmission to travel as far as possible, i.e. to separate consecutive nodes by the transmission range. Next, we determine optimal structures involving branching nodes, which are non-trivial to analyze.

A. Branching Nodes with Two Children

We first consider a branching node u with two children v_1 and w_1 as shown in Fig. 1. The distance of the first hop from u to v_1 and u to w_1 is d , where $d \leq r_t$. The first transmission from u to v and w is done by using a multicast or broadcast. The second transmissions from v_1 to v_2 and from w_1 to w_2 occur at almost the same time. The cross interference from the concurrent transmission is given by

$$\frac{P}{[r^2 + 2d(1 - \cos\theta)r + 2d^2(1 - \cos\theta)]^{\alpha/2}}.$$

Combining the received signal strength and the interference, we set SIR to SIR_{\min} and convert to decibel to get

$$0 = \left(1 - 10^{\frac{\text{SIR}_{\min}^{\text{dB}}}{5\alpha}}\right) r^2 + 2d(1 - \cos\theta)r + 2(1 - \cos\theta)d^2.$$

Solving the equation, we get $r = b_2 \cdot d$ where

$$b_2 = \frac{1 - \cos\theta + \sqrt{(\cos\theta - 1)(1 - 2 \cdot 10^{\frac{\text{SIR}_{\min}^{\text{dB}}}{5\alpha}}) + \cos\theta}}{-1 + 10^{\frac{\text{SIR}_{\min}^{\text{dB}}}{5\alpha}}}.$$

The result shows that the distance between nodes after the branching point is proportional to the distance of the first hop. Let i be the depth from u and r_i be a distance between a node at depth i and a node at depth $i + 1$. For all $i > 0$, we can consider d as a sum of all r_j , where $0 \leq j < i$, we get

$$r_i = b_2 \sum_{j=0}^{i-1} r_j = b_2(b_2 + 1)^{i-1} r_0, \text{ where } r_i \leq r_t \text{ and } r_0 \leq r_t$$

This shows that the distance between v_i and v_{i+1} grows as v_i gets further away from u until the distance reaches r_t ,

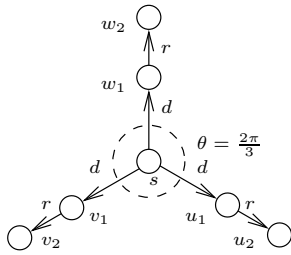


Fig. 2. A branching node s branching into three paths.

which is the transmission range and corresponds to the optimal distance for the single-path case.

B. Branching Nodes with Three Children

Next, we consider a branching node s with three children u_1 , v_1 , and w_1 as shown in Fig. 2. Applying a similar analysis to the two-child case, we get $r = b_3 \cdot d$ where

$$b_3 = \frac{3 + \sqrt{9 - 12\left(1 - 10^{\frac{10 \log 2 + \text{SIR}_{\min}^{\text{dB}}}{5\alpha}}\right)}}{2\left(10^{\frac{10 \log 2 + \text{SIR}_{\min}^{\text{dB}}}{5\alpha}} - 1\right)}.$$

Again, the distance between nodes grows as nodes get further away from the branching node, albeit in a slightly different manner. Here also, the distance will eventually reach the limit r_t .

Using the preceding analyses, we present an approximation algorithm to build a multicast tree. The algorithm applies different routing strategies depending on which is the applicable case (single path, two-way branch, or three-way branch).

C. Interference-Aware Steiner Tree Algorithm (IAST)

Since the optimal routing strategies are different for different classes, our goal is to use different routing strategies for different classes when building a multicast tree. However, one of the difficulties of applying the approach is identifying where branching should take place. Our goal is to first identify branching nodes locations and use different routing strategies for different classes when building a multicast tree.

The high level idea of the interference-aware Steiner tree routing algorithm is as follows: we are given nodes that must be connected in a multicast tree. These nodes are the source node and all the destination nodes. The first step is to identify how these nodes should be connected in a tree. The algorithm uses a Euclidean Steiner Tree approximation algorithm to locate ideal Steiner nodes in the network, using $M \cup \{s\}$ as input. We call the returned Steiner Tree a Steiner Overlay Tree since it shows the “big picture” connections between nodes in the network. An edge between two nodes in the Steiner Overlay Tree suggests that the two nodes should be connected by a path in the original graph. Note that if we view $M \cup \{s\}$ as a multicast group. The multicast group will have one Steiner Overlay Tree regardless of which node is a source node.

For each Steiner node, the algorithm finds a real node nearest to the ideal Steiner node location to act as the Steiner node. The algorithm consults the Steiner Overlay Tree to determine if the current node should be considered as the source node, a path node, or a branching node. The algorithm

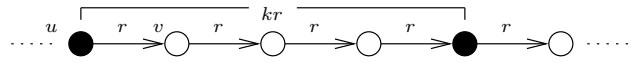


Fig. 3. An infinitely-long, equally-spaced linear network.

uses different distances for different node classes based on the analyses in Sections III-A and III-B. Note that Steiner trees contain only two-way branches and up to one three-way branch (at the source), meaning that our analyses are sufficient to handle all cases.

IV. INTERFERENCE-AWARE ROUTING FOR GENERAL MULTICAST

In this section, we consider a more general multicast scenario where the source node may begin transmission of the next packet before all receivers have received the previous packets. As a result, there may be more than one application layer packet being forwarded in the network. Multiple application layer packets being forwarded in the network means that interference in the network will be higher than the low intensity multicast case.

Since more than one application layer packet may be presented in the network, we are unable to directly apply the analyses from Section III. The extra interference means that the distances between nodes must be shortened to allow for concurrent transmissions. To tackle this problem, we introduce a scaling factor, f , to be used to scale down the distances analyzed from the Section III. Our goal is to find the most appropriate value for the scaling factor f .

A. Scaling Factor in Path Nodes

Consider a linear path in one dimension with infinite length as shown in Fig. 3. If all nodes are equally separated by a separation $r = r_t$, then all links in the path are on the edge of the SINR threshold and no concurrent transmission is possible. The schedule length in this case will be on the order of the length of the path. This phenomenon is known as the black-gray link paradox [27]. Edges with distance exactly equal to the transmission range are referred to as “black” links, and they do not allow a single concurrent transmission, no matter how distant. Edges with distance slightly below the transmission range are referred to as “gray” links, and they do allow concurrent transmission, although the allowable spatial separation might be quite large.

Given the black-gray link paradox, if the separations between nodes are scaled down to $r = f \cdot r_t$, where $0 < f < 1$, it should be possible to have multiple nodes transmit concurrently. Suppose that the schedule length is k . Two consecutive transmitters are separated by a distance $k \cdot r$. Consider a transmission from node u to node v , the total interference experienced by v is given by

$$\sum_{i=1}^{\infty} \left[\frac{P}{(ikr + r)^\alpha} + \frac{P}{(ikr - r)^\alpha} \right],$$

The SIR at the receiver v can be obtained by combining the received signal strength and the total interference at v , the SIR at v is given by the equation

$$\text{SIR}(v) = \frac{1}{\sum_{i=1}^{\infty} \left[\frac{1}{(ik+1)^\alpha} + \frac{1}{(ik-1)^\alpha} \right]}.$$

The transmission will be correctly received by v if and only if $\text{SIR}(v) \geq \text{SIR}_{\min}$; we get the following inequality:¹

$$\sum_{i=1}^{\infty} \left[\frac{1}{(ik+1)^\alpha} + \frac{1}{(ik-1)^\alpha} \right] \leq \text{SIR}_{\min}^{-1}. \quad (1)$$

We use the convergence integral test to the left hand side of (1) and get

$$\frac{1}{(k+1)^\alpha} + \frac{1}{(k-1)^\alpha} + \frac{(k+1)^{1-\alpha} + (k-1)^{1-\alpha}}{k(\alpha-1)} \leq \text{SIR}_{\min}^{-1}. \quad (2)$$

Equation (2) shows that according to SIR, the schedule length of an infinitely-long equally-spaced linear network depends only on the path-loss coefficient and the SIR_{\min} . Thus, the scaling factor can increase spatial reuse but Equation (2) shows that the schedule length will eventually converge.

For the IAST algorithm, the goal of the scaling factor is to accommodate concurrent transmissions from other nodes. We scale down the distance used when building a multicast tree by a factor f instead of using distances directly from the analyses in Section III.

Since the application of the scaling factor is not limited to IAST, we propose our second interference-aware algorithm.

B. Fixed-Distance Tree Merging Algorithm

The IAST algorithm requires global knowledge of the network to approximate a Euclidean Steiner tree and identify candidates to act as Steiner nodes. Our second algorithm, the fixed-distance tree merging (FTM) algorithm, does not require global knowledge of the network. The algorithm grows the source tree by merging the source tree with the nearest destination node until all destination nodes are connected.

The high level description of FTM algorithm is as follow. The algorithm takes a preferred scaling factor (f_p) as an input, where $0 < f_p \leq 1$. The algorithm uses the same distance $r = f_p \cdot r_t$ when building a multicast tree. For each destination node, the algorithm finds a shortest path in terms of hop counts from the receiver to all nodes in the network. The algorithm now knows the destination node nearest to the source tree.

The algorithm grows the tree from the source tree towards the destination node by using the following criteria when selecting the next hop node.

- 1) the next hop node *must* be closer to the destination node
- 2) if there exist multiple next hop nodes, the algorithm picks the next hop node that is closest to all other remaining destination nodes

After the selected destination node is merged with the source tree, the algorithm selects the next destination node nearest to the new source tree and grows the tree toward the selected node until all destination nodes are included in the

tree. In the case where node density is too low and growing the tree with $r = f_p \cdot r_t$ is not possible, FTM gradually increases r until the selected node is successfully merged or $r = r_t$.

C. Scheduling Algorithm

To accommodate the routing tree with different distances for different node classes, we propose a modified version of GreedyPhysical scheduling algorithm [28], called Tree-Based GreedyPhysical. The idea of Tree-Based GreedyPhysical is to schedule all forwarding nodes at the same depth in the same slot until no more forwarding nodes can be accommodated in the slot. Tree-Based GreedyPhysical then reverts back to the original GreedyPhysical.

V. PERFORMANCE EVALUATION

We evaluate our algorithms in three different settings. First, we start by evaluating our algorithms in a low intensity multicast setting where the analyses in Section III can be applied directly. Next, we evaluate our algorithms in a general multicast scenario where the analyses cannot be applied directly and scheduling is required. Finally, we also evaluate our algorithms in a CSMA/CA setting where nodes access the channel in a contention-based manner rather than with TDMA.

A. Simulation Parameters and Assumptions

We use ns-3.15 simulator to evaluate all algorithms. We use a physical model of 802.11g at the data rate of 6 Mbps in the simulation. All nodes use the transmission power of 40 mW and thermal noise is computed at 290K. In all simulations, 2000 nodes were uniformly distributed in a deployment area of 1000 m by 1000 m unless otherwise noted.²

We compare IAST and FTM against existing tree-based structures: MNT [15], SPT [16], and COMB [19]. For IAST algorithm, we use GeoSteiner [29] to find a Euclidean Steiner Tree. All results reported are averaged from 1000 simulations.

B. Low Intensity Multicast

We begin our evaluation with a low intensity multicast scenario. We varied the number of multicast destination nodes from 10 to 100. To prevent multiple nodes from forwarding packets at exactly the same time, we inserted random delay between 0 μs and 1000 μs before nodes forward packets to their children. We measured the delay between the time when the source node transmits the packet and the time when all destinations have received the packets. The simulation results are reported in Fig. 4.

As seen from Fig. 4, the IAST algorithm has the lowest delivery delay among all algorithms, even though SPT has the shortest distance between the source and every destination. SPT has a larger number of nodes and this creates more contention in the network, which means that nodes have to delay their transmissions when the channel is sensed as busy. MNT and COMB both have even longer delays, because they have both longer paths and higher contention.

²With 802.11g at 6 Mbps, transmission range is rather small and fairly high node density is required for the network to be connected.

¹We assume that $\text{SIR}_{\min} > 0$, where SIR_{\min} is in a linear ratio.

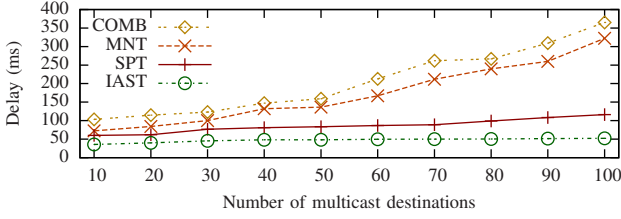


Fig. 4. End-to-end delivery delay for low-intensity multicast

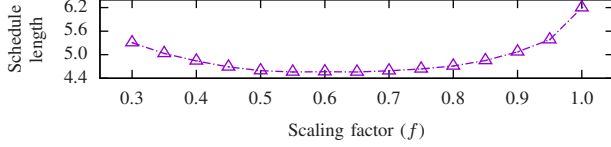


Fig. 5. IAST Schedule length vs. scaling factor

C. General Multicast

In this set of simulations, we consider a general multicast application where the application generates packets faster than the time it takes to deliver packets to all destinations. Thus, we use a scaling factor and scheduling in the algorithms.

We first evaluate the scaling factor since it is a significant parameter affecting IAST and FTM performances. In this simulation, the number of multicast destinations is fixed at 10. We vary the scaling factor from 0.3 to 1.0 and collect the schedule lengths returned by IAST algorithm. We did not use the scaling factor below 0.3 since the network became disconnected in some simulations. The simulation results are reported in Fig. 5.

Fig. 5 confirms that using different scaling factor affects IAST performance. If the scaling factor is too small, the extra nodes included in the multicast tree outweigh the gain of spatial reuse. If the scaling factor is too large, spatial reuse is not possible. However, performance is quite stable across a fairly wide range of scaling factors, e.g. 0.5 to 0.7. Based on this analysis, we have set the scaling factor to 0.7 in the remaining simulations.

We now consider the schedule length produced by different multicast routing structures, including our IAST and FTM structures. For these simulations, we varied the number of destinations from 10 to 100. There are multiple possible combinations between routing and scheduling algorithms. IAST-GP refers to IAST routing combined with the GreedyPhysical scheduling, IAST-TGP refers to IAST routing combined with Tree-Based GreedyPhysical. For MNT, SPT and COMB, we report the results from GreedyPhysical only since the difference between GreedyPhysical and Tree-Based GreedyPhysical are not statistically significant with 1000 simulations. The schedule length are reported in Fig. 6.

The results of Fig. 6 show that the schedule lengths of IAST and FTM are substantially shorter than SPT, MNT, and COMB. FTM schedules are approximately half the length of MNT schedules, 1/3 the length of COMB schedules, and less than 1/4 the length of SPT schedules for higher numbers of destinations. The shorter schedule lengths means that IAST (or FTM) can support more transmissions than other structures within the same time period. As expected, SPT and MNT

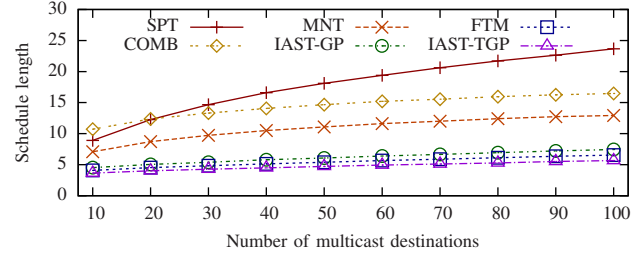


Fig. 6. Schedule length vs. number of destinations.

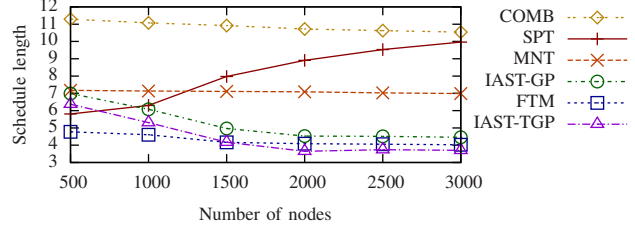


Fig. 7. Schedule length vs. node density

achieved their respective goals of shortest average path length and minimum number of forwarding nodes. However, routes built by MNT and SPT mostly consist of black links that cannot be activated concurrently with any others. As a result, most of the forwarding nodes selected by MNT and SPT must be scheduled sequentially, which increases the overall schedule length. The presence of black links is even more problematic in SPT since the number of forwarding nodes is not taken into account, resulting in a very large number of forwarding nodes that must be scheduled sequentially. For COMB structure, the rigid structure of COMB results in longer schedule length.

We also evaluate the performance of all algorithms with varying network density. We kept the number of destinations at 10 and varied the number of nodes in the network from 500 to 3000. The minimum number of nodes could not drop below 500 nodes since the network becomes disconnected in some simulations with our settings of 802.11g at 6Mbps. The schedule lengths are reported in Fig. 7.

As seen from Fig. 7, the schedule lengths of IAST and FTM algorithms increase as the node density decreases. This effect is particularly noticeable for IAST, because the closest nodes to ideal Steiner node locations can be quite far, meaning that the tree structures begin to deviate significantly from ideal Steiner trees. Since FTM is not built on Steiner trees, it is less susceptible to the lack of ideal node locations and its schedule length does not increase as dramatically for lower node densities. Even at the lowest node density, FTM's schedule length is about 17% shorter than SPT's, almost 34% lower than MNT's, and more than 55% lower than COMB's.

D. Contention-based Channel Access

Although in CSMA/CA networks, transmissions cannot be precisely scheduled, we hypothesized that the greater transmission concurrency facilitated by our interference-aware algorithms would still translate into better performance for the CSMA/CA case. To evaluate this, we performed ns-3 simulations using the standard ns-3 802.11 model. To prevent multiple nodes from forwarding packets at exactly the same

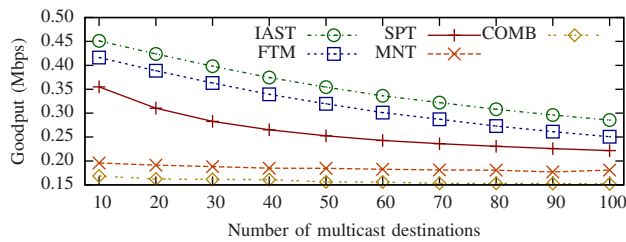


Fig. 8. Goodput of different protocols.

time, we inserted random delay before nodes forward packets to their children. The random delay was chosen uniformly between $0 \mu\text{s}$ and $1000 \mu\text{s}$. We measured the maximum goodput and report the results in Fig. 8. Note that there is no scheduling in this CSMA/CA simulation, thus, only one variation of IAST is reported.

As seen from Fig. 8, IAST and FTM still outperform SPT and MNT even without explicit scheduling. In IAST and FTM, concurrent transmission is possible while black links in SPT and MNT are intolerant of even a small interference from concurrent transmissions. The random delay helps mitigate the problem in SPT and MNT. Without the random delay, SPT and MNT performance was even lower than the reported results. Thus, our interference-aware multicast routing structures permit higher levels of concurrency and this results in significant performance benefits even without explicit scheduling.

VI. CONCLUSIONS

In this paper, we have studied the problem of multicast routing and scheduling for wireless multihop networks. We have classified nodes in multicast trees into different classes and shown by analyzing an accurate physical interference model that different classes require different routing strategies to produce optimal schedules. Based on these analyses, we have proposed two joint routing and scheduling algorithms for multicasting in wireless multihop networks. We have evaluated the performance of different algorithms through simulation and shown that the proposed algorithms outperform other algorithms in terms of schedule lengths and goodput.

VII. ACKNOWLEDGEMENTS

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REFERENCES

- [1] L. Junhai, Y. Danxia, X. Liu, and F. Mingyu, "A survey of multicast routing protocols for mobile ad-hoc networks," *Communications Surveys Tutorials*, IEEE, vol. 11, no. 1, pp. 78–91, 2009.
- [2] E. Royer and C. E. Perkins, "Multicast operation of the ad-hoc on-demand distance vector routing protocol," in *Proc. 5th Int. Conf. Mobile Computing and Networking*, 1999.
- [3] L. Ji and M. S. Corson, "Differential destination multicast—a MANET multicast routing protocol for small group," in *Proc. 20th Annu. Joint Conf. IEEE Computer and Communications Societies*, vol. 2, 2001.
- [4] A. Penttinen, "Efficient multicast tree algorithm for ad hoc networks," in *IEEE Int. Conf. Mobile Ad-hoc and Sensor Systems*, 2004, pp. 519–521.
- [5] J. Yi and C. Poellabauer, "Real-time multicast for wireless multihop networks," *Computers & Electrical Engineering*, vol. 36, no. 2, pp. 313–327, 2010.
- [6] S.-J. Lee, M. Gerla, and C.-C. Chiang, "On-demand multicast routing protocol," in *Wireless Communications and Networking Conf.*, vol. 3, 1999, pp. 1298–1302.

- [7] R. Vaishampayan and J. Garcia-Luna-Aceves, "Efficient and robust multicast routing in mobile ad hoc networks," in *IEEE Int. Conf. Mobile Ad-hoc and Sensor Systems*, Oct. 2004, pp. 304–313.
- [8] I. I. Er and W. K. Seah, "Distributed steiner-like multicast path setup for mesh-based multicast routing in ad hoc networks," in *IEEE Int. Conf. Sensor Networks, Ubiquitous, and Trustworthy Computing*, vol. 2, 2006, pp. 192–197.
- [9] D. Lun, N. Ratnakar, R. Koetter, M. Medard, E. Ahmed, and H. Lee, "Achieving minimum-cost multicast: a decentralized approach based on network coding," in *INFOCOM 2005*, vol. 3, 2005, pp. 1607–1617.
- [10] J.-S. Park, M. Gerla, D. Lun, Y. Yi, and M. Medard, "Codecast: a network-coding-based ad hoc multicast protocol," *Wireless Communications, IEEE*, vol. 13, no. 5, pp. 76–81, 2006.
- [11] J. A. Sanchez, P. M. Ruiz, and I. Stojmenovic, "GMR: Geographic multicast routing for wireless," in *3rd Annu. IEEE Communications Society Sensor and Ad Hoc Communications and Networks*, 2006.
- [12] D. Koutsonikolas, S. Das, Y. C. Hu, and I. Stojmenovic, "Hierarchical geographic multicast routing for wireless sensor networks," *Wireless Networks*, vol. 16, pp. 449–466, 2010.
- [13] C. Gui and P. Mohapatra, "Efficient overlay multicast for mobile ad hoc networks," in *IEEE Wireless Communications and Networking*, vol. 2, 2003, pp. 1118–1123.
- [14] N. Meghanathan, "Determining a sequence of stable multicast steiner trees in mobile ad hoc networks," in *Proc. 44th Annu. Southeast regional Conf.* ACM, 2006, pp. 102–106.
- [15] P. M. Ruiz and A. F. Gomez-Skarmeta, "Approximating optimal multicast trees in wireless multihop networks," in *Proc. 10th IEEE Symp. Computers and Communications*, 2005, pp. 686–691.
- [16] U. T. Nguyen, "On multicast routing in wireless mesh networks," *Computer Communications*, vol. 31, no. 7, pp. 1385–1399, 2008.
- [17] X.-Y. Li, S.-J. Tang, and O. Frieder, "Multicast capacity for large scale wireless ad hoc networks," in *Proc. 13th Annu. ACM Int. Conf. Mobile Computing and Networking*. ACM, 2007, pp. 266–277.
- [18] G. Rodolakis, A. Laouiti, P. Jacquet, and A. M. Naimi, "Multicast overlay spanning trees in ad hoc networks: Capacity bounds, protocol design and performance evaluation," *Computer Communications*, vol. 31, no. 7, pp. 1400–1412, 2008.
- [19] S. Shakkottai, X. Liu, and R. Srikant, "The multicast capacity of large multihop wireless networks," *IEEE/ACM Trans. Netw.*, vol. 18, no. 6, pp. 1691–1700, 2010.
- [20] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "Energy-efficient broadcast and multicast trees in wireless networks," *Mobile Networks and Applications*, vol. 7, no. 6, pp. 481–492, 2002.
- [21] Y. Wu, P. A. Chou, and S.-Y. Kung, "Minimum-energy multicast in mobile ad hoc networks using network coding," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1906–1918, 2005.
- [22] M. X. Cheng, J. Sun, M. Min, Y. Li, and W. Wu, "Energy-efficient broadcast and multicast routing in multihop ad hoc wireless networks," *Wireless Communications and Mobile Computing*, vol. 6, no. 2, pp. 213–223, 2006.
- [23] K. Chen and K. Nahrstedt, "Effective location-guided tree construction algorithms for small group multicast in manet," in *INFOCOM 2002*, vol. 3, 2002, pp. 1180–1189.
- [24] K. Wang, C.-F. Chiasserini, R. R. Rao, and J. G. Proakis, "A joint solution to scheduling and power control for multicasting in wireless ad hoc networks," *EURASIP Journal on Advances in Signal Processing*, vol. 2005, no. 2, 2005.
- [25] T. Erlebach and T. Grant, "Scheduling multicast transmissions under sinr constraints," in *Algorithms for Sensor Systems*, C. Scheideler, Ed. Springer Berlin Heidelberg, 2010, vol. 6451, pp. 47–61.
- [26] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.
- [27] D. M. Blough, G. Resta, and P. Santi, "Approximation algorithms for wireless link scheduling with sinr-based interference," *IEEE/ACM Trans. Netw.*, vol. 18, no. 6, pp. 1701–1712, 2010.
- [28] G. Brar, D. M. Blough, and P. Santi, "Computationally efficient scheduling with the physical interference model for throughput improvement in wireless mesh networks," *Proc. ACM Annu. Int. Conf. Mobile Computing and Networking*, pp. 2–13, 2006.
- [29] D. M. Warme, P. Winter, and M. Zachariasen, "Exact algorithms for plane steiner tree problems: A computational study," *COMBINATORIAL OPTIMIZATION-DORDRECHT*, vol. 6, pp. 81–116, 2000.