Path Selection with Amplify and Forward Relays in mmWave Backhaul Networks

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Abstract—This paper focuses on the problem of path selection with amplify-and-forward (AF) relays for long-range ultra-high-speed millimeter wave (mmWave) backhaul networks in urban environments. Relays are selected between a pair of source and destination nodes to achieve the highest signal-to-noise ratio (SNR) at the destination. We first derive an equation for the end-to-end SNR of a relay path in a setting that approximates the urban mmWave backhaul environment. Based on the derived equation, we transform the maximum throughput relay selection problem to the shortest path problem in graphs. Dijkstra’s algorithm can then be used to find maximum throughput relay paths, which however are shown to require a large number of relays. To address this, we propose a dynamic programming algorithm to find a highest throughput path with a given number of hops. Simulation results based on 3-D models of a section of downtown Atlanta show that these algorithms can be combined to find relay paths with a small number of hops and very high throughput.

I. INTRODUCTION

With the explosive growth of mobile data demand, fifth generation (5G) mobile networks with carrier frequencies in the mmWave bands are undergoing initial pilot studies. These networks will exploit the enormous amount of spectrum in these bands to increase communication capacity, potentially achieving individual link rates in the tens of Gbps. However, there are a number of fundamental differences between mmWave and existing lower-frequency communications. These include higher propagation loss, directivity, and susceptibility to blockage. Due to these issues, to achieve the promised ultra-high mmWave data rates, the communication range could be limited to a few hundred meters or less.

Relay-assisted communication is a promising solution for the ultra-high data rates required in future wireless networks. Instead of a single direct long transmission from a transmitter to a receiver, intermediate (relay) nodes can be used to enhance the diversity and reduce individual transmission length by relaying the source signal to the destination. In such a situation, the source and destination cannot communicate with each other directly because the distance between them is too long to achieve the data rate requirement and/or there are some obstacles between them preventing direct communication. The relay nodes divide the long link into some short but very high rate links which can overcome high propagation loss and sensitivity to blockage of mmWave.

The most common relay strategies are decode-and-forward (DF) and amplify-and-forward (AF). While a DF relay decodes the received signal, re-encodes it, and forwards packets toward the destination, an AF relay just amplifies its received signal and forwards it on. A DF relay’s complexity is significantly higher than an AF relay and it also requires greater computing power to perform decoding and re-encoding.

In this paper, we consider the use of AF relays in mmWave backhaul networks. Due to the very high data rates required, mmWave is considered an ideal technology for 5G wireless backhaul [1]. Several prior works have suggested using DF relay nodes to achieve backhaul data rates [2], [3] but, to our knowledge, this is the first paper to consider improving data rates by path selection with AF relays for mmWave backhaul. The precise problem we study is how to select the best relay locations to support long-range high-data-rate communications between a given source-destination pair. We first present an SNR analysis for an AF relay path with mmWave signals. We then use this analysis to transform the problem into a graph-based shortest path problem. We then apply existing graph algorithms to derive the first known efficient solutions to the optimal throughput path selection problem and the hop-constrained maximum throughput path selection problem, in a realistic wireless network setting. We also show how these algorithms can be combined to efficiently find high-throughput paths using a small number of mmWave relays.

II. RELATED WORK

End to end performance and relay path selection form the basis for our study. This section summarizes some of the relevant literature in this area. Some prior works have analyzed and optimized the performance of AF relay networks [4], [5], [6], [7], [8], [9], [10]. However, these works only considered two-hop relay networks instead of the arbitrary-hop situations that are investigated herein. In [11], the authors investigated the performance of AF relays in mmWave backhaul, but the paper focuses on two-hop relay networks, whereas we consider an arbitrary number of relays herein.

Other work has studied the performance of N-hop AF relaying systems under different situations, e.g. with or without interference, and full or half duplex communication [12], [13]. These papers focus on analysis of a given N-hop path but do not consider how to
efficiently find a best arbitrary-hop path with a given set of candidate relays, which is one of the problems we solve herein for the mmWave backhaul problem setting. We also efficiently solve the hop-constrained optimal relay path selection problem, which has not been considered in any of the prior work.

### III. Model and Problem Formulation

#### A. Network model and environment

The choice of backhaul communication technology must take into account many factors such as capacity, cost, and the need for such resources as frequency spectrum. Generally, backhaul solutions can largely be categorized into wired or wireless. Wireless backhaul is considered especially suitable for 5G networks in urban environments due to the large number of small cell base stations (BSs) that will be required for coverage. In this situation, to deploy wired connections to every small cell would incur a very high cost and might not even be possible due to accessibility issues. In this paper, we consider how to use relays to support mmWave wireless backhaul for dense small cell deployments in urban areas.

In order to achieve the high data rate requirement of mmWave links (around 10 Gbps) in backhaul networks, Line of Sight (LoS) paths have to be utilized. Particularly in urban areas, the LoS path between two BSs is often blocked due to the existence of buildings, walls, trees, and other obstacles. We use a 3D model of the environment like our earlier work [3] instead of 2D, as it gives us a more practical view of the transmission environment. Our later simulation results use an actual 3D topology of buildings in downtown Atlanta to provide a realistic evaluation environment.

#### B. AF protocol

Consider the relay fading channel from a source (s) to a destination (d) via a relay node (r), as shown in Fig. 1. Each one of them has a transmitter, a receiver and an antenna. It assumes that each station and relay node cannot transmit and receive simultaneously. The signals received by the relay node and the destination node are

\[
y_{sr} = h_{sr}x + n_t,
\]

\[
y_{rd} = \beta h_{rd}y_{sr} + n_t
\]

separately. Where \(x\) is the transmitted signal with power constraint on average transmit power \(E\{x\} \leq P_t\), \(h_{sr}\) and \(h_{rd}\) are the amplitude of the channel gain, \(n_t\) is the complex additive white Gaussian noise (AWGN) with power \(N_T\), \(\beta\) is the relay transmit average power constraint coefficient or amplification factor scaling the power transmitted by the relay. It ensures that the average transmit power at the relay \((P_r)\) is constant. Therefore, \(\beta\) can be derived as

\[
E[|\beta y_{sr}|^2] \leq P_r
\]

\[
\beta \leq \frac{P_r}{h_{sr}^2 E[|x|^2] + N_T}
\]

In our investigation, we consider \(\beta\) in terms of energy and it can be derived as

\[
\beta \leq \frac{P_r}{h_{sr}^2 E[|x|^2] + N_T}
\]

#### C. Relay fading channel model and problem formulation

Our channel model follows our earlier work on DF relays [3], which will facilitate future comparisons of AF and DF relay solutions. We use the standard assumption of additive white Gaussian noise. Link capacities are assumed to follow Shannon’s Theorem, i.e.

\[
C = B \log_2(1 + \min \{\text{SINR}, \text{SINR}_{\max}\})
\]

where \(B\) is the bandwidth of the channel in hertz, \(\text{SINR}\) is the signal to interference plus noise ratio at the receiver, and \(\text{SINR}_{\max}\) is the case of maximum capacity. For a real link, its capacity cannot be infinitely large and \(\text{SINR}_{\max}\) gives an upper bound to the capacity of the link. Therefore, the SINR can be stated as the following relationship:

\[
\text{SINR} = \frac{P_r}{N_T + I} \approx \frac{P_r}{N_T} = \text{SNR}
\]

where \(P_r\) is the power of the intended transmitter’s signal when the signal reaches the receiver, \(N_T\) is the power of thermal noise, \(I\) is the combined power of signals from any interfering transmitters and \(\text{SNR}\) is the signal to noise ratio without considering interference.

MmWave communications are generally less prone to interference, due to the directionality of transmission, which limits interference between links. The highly directional links are modeled as pseudowired in outdoor wireless mesh networks due to the narrow beamwidth of antennas [14]. The urban environment considered herein makes interference even less impactful. The relays are placed on the surfaces of different buildings with different heights. These 3-dimensional differences in relays’ positions and the narrow-beamwidth antennas reduce the likelihood that different links will align sufficiently to produce interference. Furthermore, many potentially interfering links will be blocked by large obstacles, i.e. the tall buildings present in urban settings. For these
where $i$ is the relay path in a network. In Fig. 2, different relay nodes are connected through a LoS path (and also related to the distance as the other parameters are fixed. Therefore, an intuitive conclusion would be that the shortest overall distance path will have the largest capacity. However, the analyses we perform later prove that this is not the case.

### IV. Maximum Throughput Relay Path Selection

#### A. Algorithm for Finding Maximum Throughput Relay Paths

![Diagram of relay network with transmit powers](image)

**Fig. 2.** The whole path in a relay network and transmit powers of different relay nodes

Fig. 2 shows an AF transmission model for a whole relay path in a network. In Fig. 2, $g_i$ is obtained from Eq. (8) as

$$g_i = G_t \times G_r \times \left( \frac{\lambda}{4\pi d_i} \right)^{\eta} \times e^{-\alpha d_i},$$

where $i = 1, 2, \ldots, n$ and $n-2$ is the total number of relay nodes. $\beta_i$ is the power amplification factor of each relay node from Eq. (5). The transmission power of each sender and each relay node is assumed to be the same value, $P_t$, and $N_T$ is the thermal noise power, which is the same on every link. Therefore, the SNR of the whole path can be found to be:

$$\text{SNR} = \frac{P_t g_1 g_2 \ldots g_n \beta_1 \beta_2 \ldots \beta_{n-1}}{N_T (1 + \beta_{n-1} g_n + \beta_{n-2} g_{n-1} g_n \ldots)}$$

$$= \frac{N_T (G_{n-1})}{\sum g_i^2 + \frac{N_T}{P_t} (G_{n-2}) + \ldots}$$

(10)

Where $G_{n-m}$ means the random combinations of n-3 numbers of $g_i$ added together. For example, if $n = 4$:

$$G_{n-1} = g_1 g_2 g_3 + g_1 g_2 g_4 + g_2 g_3 g_4 + g_1 g_3 g_4$$

(11)

The value of the thermal noise power, $N_T$, is approximately $10^{-12}$ and the value of $P_t = 1W$. In comparison with the value of $\frac{N_T}{P_t}$, the values of $\frac{N_T}{P_t}$, $\frac{N_T}{P_t}$, $\ldots$ can be ignored. Therefore, Eq. (10) can be simplified to:

$$\text{SNR} \approx \frac{1}{\sum \frac{1}{g_i} + \frac{1}{g_2} + \ldots + \frac{1}{g_n}}$$

(12)

Consider the following term in the denominator of Eq. (12):

$$\sum \frac{1}{g_i} = \frac{1}{g_1} + \frac{1}{g_2} + \ldots + \frac{1}{g_n}$$

(13)

The values of $N_T$ and $P_t$ are constant in this equation. Thus, the minimum value of Eq (13) will give the maximum value of SNR. This means that in order to find a relay path with maximum throughput, we need to find a relay path that minimizes Eq. (13).

We can now use Dijkstra’s shortest-path algorithm to find a maximum-throughput path. First, we build a graph with an edge for every pair of possible relay node locations that are connected with a LoS path (and also all possible source-relay and relay-destination pairs).

We can then compute the weight of every edge in the graph as the corresponding $\frac{1}{g_i}$ value. Using Dijkstra’s algorithm to find a path with the minimum sum of edge weights will then also produce a path that minimizes Eq. (13), which is a maximum-throughput path.

#### B. Numerical results and simulations

Here, we provide preliminary results to assess the kinds of relay paths that are selected by the maximum-throughput path algorithm. As mentioned earlier, we use an actual 3D topology of a section of downtown Atlanta to drive the simulations. This topology contains $277$ buildings higher than 5 meters, and each building with a height between 20 and 200 meters, one of its rooftop corners is randomly picked as a candidate BS.

1Note that this analysis matches that in [12], where the authors analyzed a more general case but also discussed the high SNR case, which roughly corresponds to the network setting we consider herein.
position (130 positions in total), BSs are expected to be deployed at positions with a good coverage of other relays mounted on the surfaces of surrounding buildings. A large number of candidate relay locations are uniformly distributed on the surfaces of each building, their locations are placed randomly on every building surface and an additional 0.002/m² candidate relay locations are uniformly distributed over all surfaces.

We set the maximum physical link distance in the simulations to be no more than 300 meters since longer LOS paths rarely exist in a dense urban environment. Similarly, considering the abundance of trees, moving vehicles and other obstacles located at relatively low heights, it is a wise choice to deploy outdoor mmWave BSs and relays at a height higher than 5 m. Otherwise, the blockage attenuation is high and temporary blockages can happen frequently. Thus, all possible LoS paths between mmWave nodes (i.e., BSs and relays) are above the level of 5 m in our simulations.

BS pairs are randomly chosen with separations in the range of [20, 200), [200, 400), [400, 600), [600, 800), and [800, 1000). 100 pairs of BSs are selected from each separation range. The maximum end-to-end throughput is computed using Dijkstra’s algorithm. The fixed values mentioned in previous equations are shown in Table I. Due to the short LOS link used in the backhaul and the high SNR at the receiver, the relatively small random attenuation due to the shadowing effect is ignored in our analysis without influencing the effectiveness. However, the implementation loss (5 dB), noise figure (5 dB), and heavy rain attenuation (10 dB/km) need to be considered in analysis. So, we include an additional link margin $L_{mi} = 10 \text{dB} + 10 \text{dB/km} \times d$ when calculating the received power. As mentioned earlier, these values are the same as those used in our earlier work [3], in order to facilitate the comparison of these works in the future research. For these values, only the beamwidth of the antenna is different from that of the earlier work. The chosen beam width of 5° is more in line with typical values of mmWave devices and allows for a more realistic evaluation of interference in later sections.

### Table I

<table>
<thead>
<tr>
<th>Parameters of Simulation Environment</th>
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</thead>
<tbody>
<tr>
<td>$BW$</td>
<td>2.16 GHz</td>
<td>$P_t$</td>
<td>1 W</td>
<td>$G_{t_e,r_x}$</td>
</tr>
<tr>
<td>$f_c$</td>
<td>60 GHz</td>
<td>$\phi$</td>
<td>$\phi$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>$L_{mi}$</td>
<td>10 dB</td>
<td>$\alpha$</td>
<td>16 dB/km</td>
<td>$L_{ma}$</td>
</tr>
</tbody>
</table>

We compare the results for three different cases. All three cases use Dijkstra’s algorithm, but each case has different edge weight values. The first case uses $\frac{1}{\phi}$ as the edge weight and will produce a maximum-throughput relay path. The second case uses the distance between two nodes as the edge weight. This case will produce a path with the shortest total distance. In the third case, we set all edge weights to be 1, which will produce a path with the minimum number of hops.

Fig. 3 compares the average throughputs and the average number of hops that are produced for these three different cases. It can be seen that, using $\frac{1}{\phi}$ as the weighted value in Dijkstra’s algorithm, the highest throughput is achieved, as expected. The throughput difference can be quite substantial for larger values of BS separation. When the BS separation is 800–1000 m, the average throughput of the maximum-throughput paths is about 9 Gbps, which is quite good. However, in this case, the maximum-throughput path uses about 28 hops on average. From the other two cases, we can see that paths with fewer than 5 hops on average exist in this situation.

![Fig. 3](image_url)

Fig. 3. Average maximum throughput and number of hops among all available paths under three different cases

The results of Fig. 3 show that, while our analysis allows us to use Dijkstra’s algorithm to find maximum-throughput paths, these paths can have a very high cost in terms of the number of relays needed. With a large number of relays, the network stability, connectivity, reliability and delay will all be negatively affected. In many situations, achieving close to maximum throughput while using a much smaller number of relays would be a preferable solution. This is the problem we consider in the next section.

V. RELAY PATH SELECTION WITH MAXIMUM HOP CONSTRAINT

### A. Hop-Constrained Relay Path Selection Algorithm

To control the number of hops in a searched path, we adopt the idea of dynamic programming (DP). We can easily find the optimal throughput path from $s$ to $d$ with at most $h$ hops, if we know the optimal throughput paths from $s$ to all neighboring nodes of $d$ and $d$ itself with a maximum $(h - 1)$ hop constraint.

Algorithm 1 shows the pseudo-code for our improved path selection algorithm, which does a search of possible maximum throughput paths with a limited number of hops. The pseudocode uses a dynamic programming approach with recursion. Since best paths found with shorter hop counts are recorded as the algorithm executes and these values can be looked up later in constant time without additional recursion, the time complexity
of the algorithm is $O(hN^2)$, where $h$ is the maximum hop count and $N$ is the number of nodes in the graph.

Algorithm 1 Finding the path with maximum throughput using a limited number of hops

Input: $s$ (source), $d$ (destination), $V$ (nodes), $N$ (neighbor map), $h$ (max hop), $W$ (weight matrix)

Output: path

1: Initialize $|V| \times (h + 1)$ matrix dist to Inf;
2: dist[s][0] = 0; // distance from $s$ to $s$ at 0 hop is 0.
3: Initialize $|V| \times (h + 1)$ matrix pre to -1;
4: pre[s][0] = s; // pre node from $s$ to $s$ at 0 hop is $s$.
5: minDist = findPath(d, h, dist, pre);
6: if minDist == Inf then
7: return NULL; // no path found
8: else
9: return pathRecovery(d, h, pre);
10: Function findPath(d′, h′, dist, pre)
11: if $h' == 0$ then
12: return dist[d′][0];
13: if dist[d′][h′] < Inf then
14: return dist[d′][h′];
15: $N_{d'} = N.get(d')$; // store neighbors of $d'$ in $N_{d'}$
16: for n in $N_{d'}$ do
17: temp = findPath(n, h′ − 1, dist, pre) + $W[n][d']$;
18: if temp < dist[d′][h′] then
19: dist[d′][h′] = temp;
20: pre[d′][h′] = n;
21: findPath(d′, h′ − 1, dist, pre);
22: if dist[d′][h′ − 1] < dist[d′][h′] then
23: dist[d′][h′] = dist[d′][h′ − 1];
24: pre[d′][h′] = pre[d′][h′ − 1];
25: return dist[d′][h′];
26: Function pathRecovery(d′, h′, pre)
27: cur = $d'$
28: while cur ≠ $s$ do
29: path.add(cur);
30: cur = pre[|cur|][h′ − 1];
31: path.add(cur);
32: return path;

We first discuss one representative example to illustrate how throughput varies with hop count. Fig. 4 shows the results for this example, which is a pair of base stations in the separation range of (600, 800) meters. Note that throughput increases rapidly as hop count is increased beyond the minimum value and then increases only gradually up to the maximum throughput of 9.501 Gbps, which occurs at a hop count of 24. With a hop count of only 8, a throughput of 8.673 Gbps is achieved, which is already more than 90% of the maximum.

We also evaluate the combination algorithm in aggregate using the same 100 pairs of BSs in the five different ranges of distances as described in Section IV.B. Here, we set the throughput target to be 90% of the maximum throughput. Fig. 5 shows that the number of hops is significantly reduced with only a 10% reduction in throughput. For example, in the 800m to 1000m BS separation case, the average number of hops is reduced from about 28 to about 8, which represents a substantial savings in the number of relays, a corresponding reduction in delay, and other aforementioned benefits.

VI. INTERFERENCE CONSIDERATIONS

The results presented in the previous sections assume that self-interference does not occur along paths. If this assumption is violated, the actual throughput will be lower than our analyses predict. For the same data sets used in previous sections (100 BS pairs for each BS separation range), Table II shows the frequency of interference occurring on the minimum hop, maximum
throughput, and 90% of maximum throughput paths. The table shows that interference is rare. For example, on the 90% of maximum throughput paths, there are only 10 paths out of 500 total that experience interference.

**TABLE II**

<table>
<thead>
<tr>
<th>BS distance (m)</th>
<th>Interference on optimal path</th>
<th>Interference on path with 90% throughput</th>
<th>Interference on path with min. hops</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20, 200)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>(200, 400)</td>
<td>3%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>(400, 600)</td>
<td>2%</td>
<td>1%</td>
<td>0%</td>
</tr>
<tr>
<td>(600, 800)</td>
<td>1%</td>
<td>3%</td>
<td>0%</td>
</tr>
<tr>
<td>(800, 1000)</td>
<td>5%</td>
<td>6%</td>
<td>0%</td>
</tr>
</tbody>
</table>

While the interference probability is small, our approach does occasionally choose a path with interference. In this situation, we want to be able to find a different non-interfering path with close to the same performance (throughput and hop count) as the interfering path. We therefore investigated a simple method to guarantee interference-free paths. In this method, if a path is chosen that has interference between links, we pick any one of the links experiencing interference and give it a very high weight value. We then re-run the algorithm to find a new path, which will not contain that link. We repeat this procedure as many times as necessary until an interference-free path is found.

We compared the newly found interference-free paths using this method with the previous paths, using 90% of maximum throughput paths as examples. From Table II, the previous paths with 90% of maximum throughput in [400, 600] distance range, interference occurred in one pair of BSs out of 100 total pairs. Comparing this one path with the interference-free path we found using our method, the hop count of the interference-free path increased from 3 to 6. In this case, the average number of hops for these 100 pairs of BSs (compare to Fig. 5) only increased from 4.61 to 4.63. For the distance ranges of [600, 800) and [800, 1000), there are three pairs and six pairs of BSs that have interference in the paths, respectively. For the interference-free paths found by our method, two of the three pairs hop counts increased by 1 and two of the six pairs hop counts increased by 1, while the other paths’ hop counts did not increase at all. The average number of hops for the [600, 800) case increased from 6.54 to 6.56 and for the [800, 1000) case, it increased from 8.68 to 8.70.

In summary, our approach can easily be modified to only produce interference-free paths, with minimal impact on the average throughput and hop count.

**VII. CONCLUSION**

In this paper, we have presented path selection algorithms to find high-throughput paths using amplify-and-forward relays to support mmWave backhaul networks. While we can very efficiently find maximum-throughput paths, they often require a very large number of relays. With a slightly less efficient algorithm, however, we were able to find high-throughput paths that use far fewer relays. We also verified that the paths produced by our algorithms have a very high likelihood of being interference free and we presented an extended approach that handles the rare interference cases.

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**REFERENCES**


