ECE 3020: Midterm 2

Instructions: You have 50 minutes to complete this test. The test is closed book and closed notes. No calculators are allowed. Make sure to show your work on all problems. No credit will be given for answers without sufficient work.

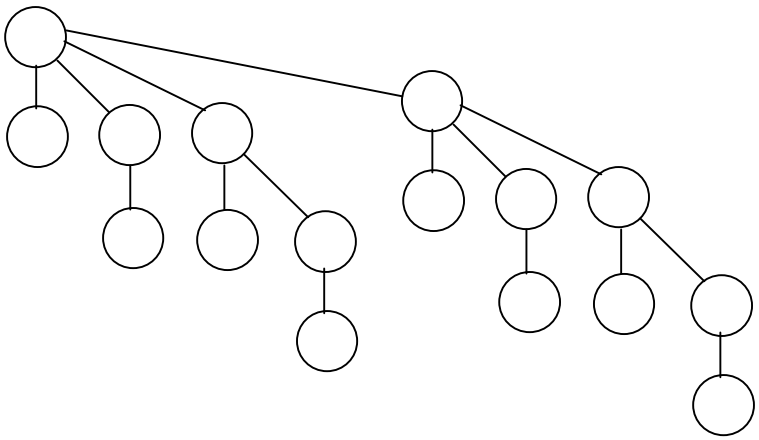
Problem 1	_____
Problem 2	_____
Problem 3	_____
Problem 4	_____
TOTAL	_____

1) 25 points

Sheryl needs to choose a 12-character password made up only of lower-case and upper-case letters and digits. (Assume a lower-case letter is different from the corresponding upper-case one.) For the system to allow a password to be used, it must contain at least one lower-case letter, at least one upper-case letter, *and* at least one digit. If Sheryl writes a program to randomly generate 12-character passwords made up of the possible characters, what is the probability that the first password her program generates is allowable? Assume all 12-character passwords made up of the possible characters are equally likely to be generated by Sheryl's program.

2) 25 points

A binomial tree of order k is recursively defined as follows. A binomial tree of order 0 is a single node. A binomial tree of order $k > 0$ consists of a root node and k children, where the i^{th} child of the root is the root of a binomial tree of order i, $0 \leq i \leq k\text{-}1$. A drawing of a binomial tree of order 4 is:



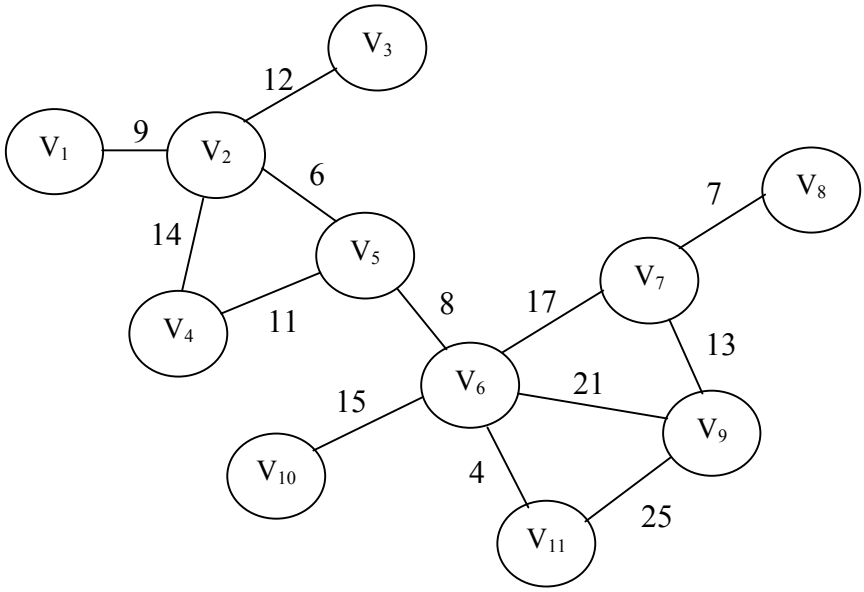
Prove, using structural induction, that the number of nodes in a binomial tree of order k is 2^k. You ***must*** use structural induction to receive full credit for this problem.

3) 25 points

Consider the graph shown below.

a) How many different spanning trees are there for this graph? Justify your answer.

b) Using v_1 as the starting node, execute Prim's Algorithm to find a minimum-weight spanning tree of the graph. Show the edges of the minimum-weight spanning tree ***in the order that they are added*** by the algorithm.



4) 25 points

a) Use the Ford-Fulkerson Algorithm to find a flow function that maximizes the flow in the below network having the capacities shown and calculate the total flow that results. List the augmenting paths that you use to get your solution in order to get full credit.

b) Show a minimum cut in the network whose capacity is equal to the maximum flow you found in a).