ECE 3020: Midterm I

Instructions: You have 50 minutes to complete this test. The test is closed book and closed notes. No calculators are allowed. Make sure to show your work on all problems. No credit will be given for answers without sufficient work.

Problem 1
Problem 2
Problem 3
Problem 4

TOTAL
1) 25 points
Solve the following recurrence equation exactly: \( T(n) = 3T(n/3) + an, T(3) = b \)
2) 25 points

Consider the recurrence equation: \( T(n) = T(n-1) + T(n-2) + b, \ T(0) = T(1) = a \)

Show, using proof by induction, that \( T(n) \leq c \cdot 2^n \), \( \forall \ n \geq 0 \). As part of the proof, derive conditions on \( c \) that allow the proof to work.
3) 25 points
A survey asked 100 Georgia Tech students whether they had taken courses in the following subjects: Math (M), Computer Engineering (C), and Business (B). The following results were obtained (where, for example MC means that students had taken both Math and Computer Engineering courses):

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>C</th>
<th>B</th>
<th>MC</th>
<th>MB</th>
<th>CB</th>
<th>MCB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80</td>
<td>30</td>
<td>35</td>
<td>25</td>
<td>25</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

a) How many students surveyed did not take courses in any of the 3 subjects?

b) How many students took a course in Computer Engineering but not in either of the other two areas?
4) 25 points

The Ackermann function is defined recursively as follows:

\[ A(0, y) = y + 1 \]

\[ A(x, 0) = A(x-1, 1) \text{ if } x > 0 \]

\[ A(x, y) = A(x-1, A(x, y-1)) \text{ if } x, y > 0 \]

Write pseudocode for a recursive function to calculate \( A(x, y) \) for \( x, y \geq 0 \).