Theorem: A tree with \( n \) nodes has \( n-1 \) edges.

Proof: (using structural induction)

Base case: A single node is a tree. It has 0 edges. \( \checkmark \)

Inductive step:

Consider an arbitrary tree \( T \) with \( n > 1 \) nodes.

Assume all trees with \( k \leq n \) nodes have \( k-1 \) edges.

By the recursive def'n of a tree, \( T \) can be drawn as:

\[
\begin{array}{c}
\text{T} \\
\text{r} \\
\text{T}_i \\
\text{. . .} \\
\text{T}_m
\end{array}
\]

where \( m \geq 1 \) is the number of children of \( r \) in \( T \).

Since each \( T_i \) has \( \leq n \) nodes (fewer nodes than \( T \)), the inductive hypothesis holds for each \( T_i \).

Let \( n_i \) be the number of nodes in \( T_i \). Then, the number of edges in \( T_i = n_i - 1 \) (by inductive hypothesis).

The total number of edges in \( T \)

\[
\sum_{i=1}^{m} (n_i - 1) + m = \sum_{i=1}^{m} n_i - m + m = n - 1, \text{ QED}
\]

because subtrees together contain all nodes in \( T \) except one (the root).