Theorem: A tree with n nodes has $n-1$ edges.

Proof: (using structural induction)

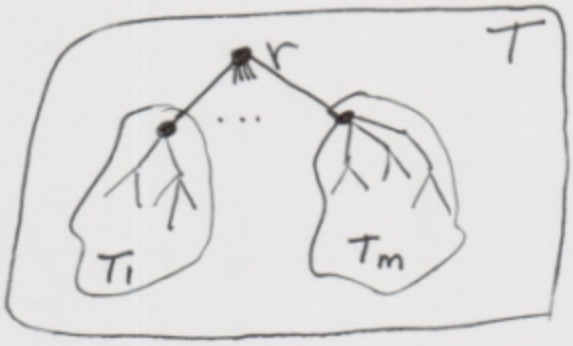
Base case: A single node is a tree. It has 0 edges. ✓

Inductive step:

Consider an arbitrary tree T with $n > 1$ nodes.

Inductive hypothesis: Assume all trees with $k < n$ nodes have $k-1$ edges.

By the recursive def'n of a tree, T can be drawn as:



where $m \geq 1$ is the number of children of r in T

Since each T_i has $< n$ nodes (fewer nodes than T), the inductive hypothesis holds for each T_i.

Let n_i be the number of nodes in T_i. Then, the number of edges in $T_i = n_i - 1$ (by inductive hypothesis)

The total number of edges in T

$$= \underbrace{\sum_{i=1}^{m} (n_i - 1)}_{\text{edges in the subtrees}} + \underbrace{m}_{\text{edges connecting subtrees to } r} = \sum_{i=1}^{m} n_i - \cancel{m} + \cancel{m} = n-1, \text{ QED}$$

because subtrees together contain all nodes in T except one (the root)