total number of sequences $= 2^7 = 128$; each sequence is equally likely, so we can count outcomes (sequences) to find probabilities

(a) $P(\text{winning | W in 1st position}) = \frac{P(\text{winning and W in 1st position})}{P(\text{W in 1st position})}$

$P(\text{W in 1st position}) = \frac{1}{2}$

$P(\text{winning and W in 1st position}) = \frac{\binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}}{128}$

$= \frac{20 + 15 + 6 + 1}{128} = \frac{42}{128}$

$\therefore P(\text{winning | W in 1st position}) = \frac{42}{128} \times \frac{1}{2} = \frac{21}{64} = 0.328125$

(b) similar to (a)

$P(\text{W in 1st 2 positions}) = \frac{1}{4}$

$P(\text{winning and W in 1st 2 positions}) = \frac{\binom{5}{3} + \binom{5}{4} + \binom{5}{5}}{128}$

$= \frac{10 + 10 + 5 + 1}{128} = \frac{26}{128}$

$\therefore P(\text{winning | W in 1st 2 positions}) = \frac{26}{128} \times \frac{1}{4} = \frac{26}{32} = 0.8125$
can be solved similarly to (a), (b) however, a simpler solution is to simply evaluate the last 4 games as a new random experiment where the team must win at least 2 games.

\[ P(\text{winning} \mid \text{Win 2 of 1st 3 positions}) \]

\[ = \frac{(4) + (4) + (4)}{2^4} \]

\[ = \frac{11}{16} = 0.6875 \]

(2) Let us use the Law of Total Probability and condition on different possible outcomes of the 1st 2 rolls.

something < 50 on 1st 2 rolls cannot yield \( \geq 100 \) overall, so we will lump those cases together.

\[ P(\geq 100 \text{ overall}) \]

\[ = P(\geq 100 \mid < 50 \text{ on 1st 2}) \cdot P(< 50 \text{ on 1st 2}) \]

\[ + P(\geq 100 \mid 50 \text{ on 1st 2}) \cdot P(50 \text{ on 1st 2}) \]

\[ + P(\geq 100 \mid 60 \text{ on 1st 2}) \cdot P(60 \text{ on 1st 2}) \]

\[ + \ldots + P(\geq 100 \mid 100 \text{ on 1st 2}) \cdot P(100 \text{ on 1st 2}) \]
\[\begin{align*}
\text{Given:} & \quad P(\text{50 on 3}\text{rd} ) \cdot P( (40,40), (30,30), (40,20) \text{or} (40,10) \text{ on 1st 2}) \\
& + P(\text{40 or 50 on 3}\text{rd}) \cdot P( (40,50), (20,40), (30,30), (40,20), (50,10) \text{ on 1st 2}) \\
& + P(\text{40,40,50 on 3}\text{rd}) \cdot P( (20,50), (30,40), (40,30), (50,20) \text{ on 1st 2}) \\
& + P(20,30,40,50 \text{ on 3}\text{rd}) \cdot P( (30,50), (40,40), (50,30) \text{ on 1st 2}) \\
& = 0.02 \times (2 \times 0.5 \times 0.08 + 2 \times 0.25 \times 0.15) \\
& \quad + 0.1 \times (2 \times 0.5 \times 0.02 + 2 \times 0.25 \times 0.08 + 0.15^2) \\
& \quad + 0.25 \times (2 \times 0.25 \times 0.02 + 2 \times 0.15 \times 0.08) \\
& \quad + 0.5 \times (2 \times 0.15 \times 0.02 + 0.08^2) \\
& \quad + 4 \times (2 \times 0.08 \times 0.02) + 1 \times 0.02^2 \\
& = 0.0031 + 0.00825 + 0.0085 + 0.0062 + 0.0032 + 0.0004 \\
& = 0.02965
\end{align*}\]

\[\boxed{0.02965}\]

(b) \quad \text{In (a), we calculated } P(\geq 100 \text{ overall}) \text{ if the 2 events are independent, then } P(\geq 100 \text{ overall}) = P(\geq 100 \text{ overall} | 1\text{st roll is 30}) \text{. So, calculate } P(\geq 100 \text{ overall} | 1\text{st roll is 30}) \text{ and compare with answer to (a).}
\[ P(\geq 100 \text{ overall} \mid \text{1st roll is 30}) \]

\[ = (P \geq 70 \text{ on last 2 rolls}) \]

\[ = P((30,40),(40,30),(40,40),(30,50),(50,30),(40,50), \]
\[ (50,40), \text{ or } (50,50) \text{ on last 2 rolls}) \]

\[ = 2 \times 0.15 \times 0.08 + 0.08^2 + 2 \times 0.02 \times 0.15 + 2 \times 0.08 \times 0.02 + 0.02^2 \]

\[ = 0.024 + 0.0064 + 0.006 + 0.0032 + 0.0004 \]

\[ = 0.04 \]

Since \( P(\geq 100 \text{ overall}) \neq P(\geq 100 \text{ overall} \mid \text{1st roll is 30}) \)

the events \( \{ \geq 100 \text{ overall} \} \) and \( \{ \text{1st roll is 30} \} \)

are \underline{not independent}.

\(\text{(C)}\) for expected total score, we use linearity of expectation

\[ E[\text{total score}] = 3 \times E[\text{score on 1 roll}] \]

\[ = 3 \times (10 \times 0.15 + 20 \times 0.25 + 30 \times 0.15 + 40 \times 0.08 + 50 \times 0.02) \]

\[ = 3 \times 18.7 = \boxed{56.1} \]

\[ \text{Var}[\text{1 roll}] = E[(\text{score on 1 roll})^2] - (E[\text{score on 1 roll}])^2 \]

\[ = 0.25 \times 100 + 0.25 \times 400 + 0.15 \times 900 + 0.08 \times 1600 + 0.02 \times 2500 \]

\[ - (18.7)^2 \approx \boxed{113.3} \]
(3) a) Use Law of Total Probability

\[ P(\text{younger than 19}) = P(\text{younger than 19} | \text{female}) \cdot P(\text{female}) + P(\text{younger than 19} | \text{male}) \cdot P(\text{male}) \]

\[ = 0.92 \times 0.53 + 0.79 \times 0.47 \]

\[ = 0.8589 \]

b) From a), \( P(\text{younger than 19}) \neq P(\text{younger than 19} | \text{female}) \)

\[ \therefore \ \text{age and gender are not independent} \]

c) Use Bayes' Theorem

\[ P(\text{male} | \geq 19 \text{ years old}) = \frac{P(\geq 19 \text{ years old} | \text{male}) \cdot P(\text{male})}{P(\geq 19 | \text{male}) \cdot P(\text{male}) + P(\geq 19 | \text{female}) \cdot P(\text{female})} \]

\[ = \frac{0.21 \times 0.47}{0.21 \times 0.47 + 0.08 \times 0.53} \]

\[ \approx 0.6995 \]

d) Again, use Bayes' Theorem

\[ P(\text{female} | \text{younger than 19}) \]

\[ = \frac{P(\text{younger than 19} | \text{female}) \cdot P(\text{female})}{P(\leq 19 | \text{female}) \cdot P(\text{female}) + P(\leq 19 | \text{male}) \cdot P(\text{male})} \]

\[ = \frac{0.92 \times 0.53}{0.92 \times 0.53 + 0.79 \times 0.47} \]

\[ \approx 0.5677 \]