1) Use overcount, then exclude.

Count total number of outcomes and subtract the number that have zero 1's.

Total number of outcomes = 56 (from book Ex. 4.16) to have zero 1's is like having dice with only 5 values (2, 3, 4, 5, 6).

*: Distribute the 0 3 objects (dice) into 5 bins (dice values) instead of 6.

Number of outcomes with zero 1's = \( \binom{5+3-1}{3} \)

\[ = \binom{7}{3} = 35 \]

Number of outcomes with at least one 1

\[ = 56 - 35 = \square 21 \]

2a) Use sequence of choices.

Pick suit w/ 4 cards \(-\binom{4}{1} = 4\)

Pick cards of "4-suit" \(-\binom{13}{4} = 715\)

Since all other suits have 3 cards, we do not have to pick any more suits.
for each "3-suit", pick 3 cards \[ \binom{13}{3} \times \binom{13}{3} \times \binom{13}{3} \]
\[ = 286^3 \]

**total number of combinations**

\[ = 4 \times 715 \times 286^3 = \boxed{66,905,856,160} \]

(b) pick suits for 5,4,3,1 cards \[ = 4 \times 3 \times 2 \times 1 = 24 \]
pick cards of "5-suit" \[ = \binom{13}{5} = 1,287 \]
pick cards of "4-suit" \[ = \binom{13}{4} = 715 \]
pick cards of "3-suit" \[ = \binom{13}{3} = 286 \]
pick cards of "1-suit" \[ = \binom{13}{1} = 13 \]

**total no. of combinations**

\[ = 24 \times 1,287 \times 715 \times 286 \times 13 \]
\[ = \boxed{82,111,732,560} \]

(c) pick suits \[ = 4 \times 3 = 12 \]
pick cards - same as (a),(b)

**total number**

\[ = 12 \times \binom{13}{4} \times \binom{13}{4} \times \binom{13}{3} \times \binom{13}{2} \]
\[ = \boxed{136,852,887,600} \]

(d) **total no.**

\[ = 12 \times \binom{13}{4} \times \binom{13}{2} \times \binom{13}{2} \times \binom{13}{0} \]
\[ = \boxed{52,200,720} \]
3. \( E_1 = \{ \text{numbers divisible by } 3 \} \) \( |E_1| = 333 \)
\( E_2 = \{ \text{numbers divisible by } 5 \} \) \( |E_2| = 200 \)
\( E_3 = \{ \text{numbers divisible by } 7 \} \) \( |E_3| = 142 \)

\( E_1 \cap E_2 = \{ \text{numbers divisible by } 15 \} \) \( |E_1 \cap E_2| = 66 \)

\( E_1 \cap E_3 = \{ \text{numbers divisible by } 21 \} \) \( |E_1 \cap E_3| = 47 \)

\( E_2 \cap E_3 = \{ \text{numbers divisible by } 35 \} \) \( |E_2 \cap E_3| = 28 \)

\( E_1 \cap E_2 \cap E_3 = \{ \text{numbers divisible by } 105 \} \) \( |E_1 \cap E_2 \cap E_3| = 9 \)

by inclusion-exclusion principle:

\( |E_1 \cup E_2 \cup E_3| = 333 + 200 + 142 - 66 - 47 - 28 + 9 \)

\[ = 543 \]

numbers divisible by 3 or 5 or 7

\[ \left| \{ \text{numbers not divisible by } 3, 5, \text{ or } 7 \} \right| \]

\[ = 1000 - 543 \]

\[ = \boxed{457} \]