Inclusion-Exclusion Principle for Counting

to count when there are multiple overlapping sub-cases

sub-cases $N_1, N_2, \ldots, N_n$

$$|N_1 \cup N_2 \cup \cdots \cup N_n| = \sum_{i=1}^{n} |N_i| - \sum_{\text{all pairs } i,j} |N_i \cap N_j| + \sum_{\text{all triples } i,j,k} |N_i \cap N_j \cap N_k| - \sum_{\text{all } i,j,k,l} |N_i \cap N_j \cap N_k \cap N_l| + \cdots + (-1)^{n-1} |N_1 \cap N_2 \cap \cdots \cap N_n|$$

When $n = 3$, this becomes:

$$|N_1 \cup N_2 \cup N_3| = |N_1| + |N_2| + |N_3| - |N_1 \cap N_2| - |N_1 \cap N_3| - |N_2 \cap N_3| + |N_1 \cap N_2 \cap N_3|$$
a group of ice cream lovers is surveyed and asked which flavors they like out of chocolate, vanilla, strawberry.

It's OK to like more than 1 flavor, i.e. choices are not disjoint.

Assume everyone likes at least 1 of the 3.

Results:
(E1) like chocolate: 20
(E2) like vanilla: 15
(E3) like strawberry: 7
(E1 ∩ E2) like choc/vanilla: 12
(E1 ∩ E3) like choc/straw: 4
(E2 ∩ E3) like van/straw: 3
(E1 ∩ E2 ∩ E3) like all 3: 2

How many people were surveyed?

|E1 U E2 U E3| = |E1| + |E2| + |E3| - |E1 ∩ E2| - |E1 ∩ E3| - |E2 ∩ E3| + |E1 ∩ E2 ∩ E3|

= 20 + 15 + 7 - 12 - 4 - 3 + 2 = 25

(if we didn't assume everyone liked at least one, need another entry "likes none"