Inclusion-Exclusion for Counting

A group of ice cream lovers is surveyed and asked which flavors they like out of chocolate, vanilla, strawberry.

It's OK to like more than 1 flavor, i.e. choices are not disjoint.

Assume everyone likes at least 1 of the 3.

Results:

- (E₁) like chocolate: 20
- (E₂) like vanilla: 15
- (E₃) like strawberry: 7
- (E₁ ∩ E₂) like choc/vanilla: 12
- (E₁ ∩ E₃) like choc/straw: 4
- (E₂ ∩ E₃) like van/straw: 3
- (E₁ ∩ E₂ ∩ E₃) like all 3: 2

How many people were surveyed?

\[ |E₁ ∪ E₂ ∪ E₃| = |E₁| + |E₂| + |E₃| - |E₁ ∩ E₂| - |E₁ ∩ E₃| - |E₂ ∩ E₃| + |E₁ ∩ E₂ ∩ E₃| \]

\[ = 20 + 15 + 7 - 12 - 4 - 3 + 2 = 25 \]

(If we didn't assume everyone liked at least one, need another entry: "likes none" would need to be added.)