ECE 3803: Final Exam

Instructions: You have 2 hours and 50 minutes to complete this exam. The exam is closed book and closed notes, with the exception of one 8.5” x 11” sheet of paper. No calculators are allowed. Make sure to show your work on all problems. No credit will be given for answers without sufficient work.

Problem 1	_________
Problem 2	_________
Problem 3	_________
Problem 4	_________
Problem 5	_________
Problem 6	_________
Problem 7	_________
Problem 8	_________
TOTAL	_________

1) 12 points

Use proof by induction to prove that $(n+1)^2 < 2n^2$, for all $n \geq 3$.

2) 13 points

Write C-like or Java-like pseudocode to implement a recursive sorting function that operates in the following way. When given an array of n integers to sort, the function ignores the first element of the array, recursively sorts the remaining n-1 elements, and then goes through the array to place the first element in its proper place shifting the other elements to the left as appropriate.

3) 12 points

a) How many 3-of-a-kind poker hands are there? A poker hand consists of 5 cards. A 3-of-a-kind hand has 3 cards of the same rank. Make sure not to count 4-of-a-kind hands and full-house hands. A full house is made up of a 3-of-a-kind and a pair together.

b) What is the probability of being dealt a 3-of-a-kind hand when 5 cards are dealt from a 52-card deck? Assume that all hands are equally likely to be dealt.

4) 13 points

A complete ternary tree is a tree where every non-leaf node has exactly 3 children and all leaves are at the same depth. Prove that a complete ternary tree of height h has exactly $(3^{h+1} - 1)/2$ nodes.

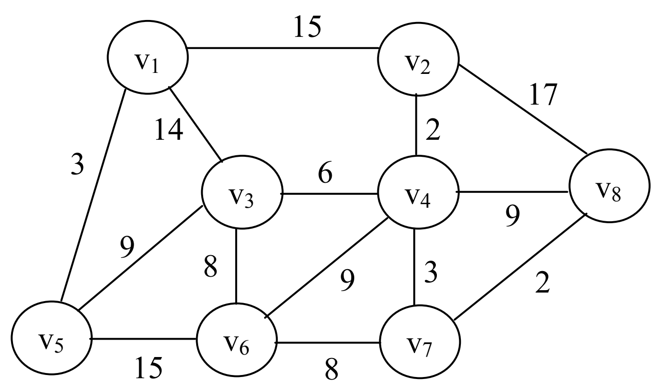
5) 12 points

a) Draw the heap that results after the following items are inserted in sequence into an empty heap: 36, 15, 9, 24, 17, 21, 55, 12, 34. Assume the larger the value of the item, the higher is its priority.

b) Starting from your answer to a), show the resulting heap after 3 items are removed

6) 13 points

Use Prim's Algorithm and Dijkstra's Algorithm to find a minimum spanning tree and shortest path tree for the below graph. For Dijkstra's Algorithm, use v_6 as the source node.



7) 12 points

Reduce the following state machine to a minimum number of states:

Current State	Next State		Output	
	X = 0	X = 1	X = 0	X = 1
A	H	C	1	0
B	C	D	0	1
C	H	B	0	0
D	F	H	0	0
E	C	F	0	1
F	F	G	0	0
G	G	C	1	0
H	A	C	1	0

8) 13 points

Draw a parse tree for the expression “w c w c { s ; s ; w c s ; }” with the following context-free grammar:

<S> → w c <S>

<S> → { <L> }

<S> → s ;

<L> → <L> <S>

<L> → ε